Optimal control of the temperature in a solar furnace

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SUMMARY

This article describes the application of optimal control to a solar furnace that is used to perform temperature stress cycling tests in material samples. This process is characterized by having nonlinearities that depend on the sample properties and relate the temperature of the sample with the solar energy fluctuations and the position of the furnace shutter. An optimal control problem with fixed terminal time and free terminal state and control constraints is addressed in continuous time domain. The solution is approximated using discretized state and costate equations and applied to the furnace according to a receding horizon strategy. The performance of the overall system is evaluated from computer simulations which show that the controller is able to tolerate, up to some degree, the presence of parameter uncertainty.

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1. INTRODUCTION

Solar furnaces, such as the vertical one shown in Figure 1, concentrate solar energy in a focus and are often used to perform material tests, [1–3].

The Solar Energy Laboratory (Odeillo, Southern France, a facility of PROMES, CNRS) and the Plataforma Solar de Almeria (southern Spain) are two sites where solar furnaces are used for such purpose. Experimental results obtained with the solar furnace of Odeillo, considered in this paper, are described in [4, 5], where a PI controller is used to command the shutter position as a function of the temperature tracking error. An enhanced temperature control architecture was proposed in [6, 7], based on adaptive control, and in [8], where a cascade control architecture with two loops was used to decouple the shutter nonlinearity from the temperature dynamics. Other works developed at the Plataforma Solar de Almeria address the control of solar furnaces by including constrained temperature control and disturbance rejection [9] [10], and linearization with the Generalized Predictive Control (GPC) algorithm [11].

According to a cascade architecture, shown in Figure 2, the inner control loop controls the shutter, to compensate its nonlinear static behavior, and its reference is used to compensate the effects of the sun power variability caused by atmospheric perturbations such as clouds.

In this article, the temperature controller is designed using optimal control and Pontryagin’s maximum principle. The contribution consists of exploring the application of optimal control, combined with a receding horizon strategy to the temperature control of solar furnaces. A nominal nonlinear model of the temperature process is used to design the optimal controller that is developed in continuous time.

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Figure 1. Schematic view of the vertical solar furnace of Odeillo PROMES laboratory considered in this article.

Figure 2. Control system architecture for the solar furnace based on a cascade structure.

The remaining part of this article is organized as follows. Section 2 describes the dynamic model of the solar furnace. The design methodology is presented in Section 3. The evaluation of the control methodology using computer simulations is presented in Section 4. Conclusions are drawn at the final section.

2. THE SOLAR FURNACE PROCESS

As sketched in Figure 1, a solar furnace system comprises a heliostat mirror that tracks the sun and guides the sunlight to a parabolic mirror, a parabolic mirror that concentrates the energy of the sun at the focus, and a shutter that is used to manipulate the amount of energy that arrives at the focus by moving a set of blades. Figure 3(a) and (b) shows examples of solar furnaces with shutters of different geometries.‡

Therefore, the solar furnace model comprises two dynamic models: a dynamic model that describes the interactions between the power at the focus and the temperature of the sample, and another one that describes the shutters dynamics. These models are described hereafter.

2.1. Shutter model

Figure 3 shows two examples of vertical solar furnaces, with different types of shutters. The furnace of Figure 3(a) is the one considered in this article. The shutter operates in closed loop using a servo mechanism. In this work, it is assumed that the shutter has a dynamics that is much faster than the one of the thermal closed-loop subsystem [4, 8], and thus, only the static function of the shutter is considered. This function described by

$$s_{fs}(u_s(t)) = 1 - \frac{\cos(\theta_0 + u_s(t)(90^\circ - \theta_0))/100}{\cos(\theta_0)}.$$ 

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Figure 3. Examples of solar furnaces with different shutters (located after and before the concentrating flux) that are used to adjust the amount of power that is applied to the samples being tested.

where \( s_{fs}(.) \) is the nonlinear static function of the shutter that gives the percentage of power applied to the sample, \( u_s(.) \) is the shutter command that satisfies \( 0 \leq u_s(t) \leq 100 \), and \( \theta_0 = 25^\circ \). The control subsystem of the shutter compensates the inertia and the nonlinear effects of the blades, and it is able to move them to the target angle in less than 0.5 s, comprising the time required to drive the shutter from the closed position to the fully open position. The maximum angular speed of the blades occurs at half of this range.

2.2. Temperature model of the sample

As in [1], an energy balance is used to describe the temperature model of the sample. The sample is assumed to have a circular shape, with 2 cm diameter and 2 mm height. The temperature of the sample \( T_s(t) [K] \) verifies

\[
\frac{dT_s(t)}{dt} = -\alpha_1 [T_s^4(t) - T_e^4(t)] - \alpha_2[T_s(t) - T_e(t)] + \alpha_3 G_s(t) s_{fs}(u_s(t)).
\]  

(2)

Here, \( T_e [K] \) represents the temperature of the environment that contributes to losses by radiation and convection. The function \( G_s(t) \) represents the solar flux, and \( s_{fs} \) is defined in (1). The terms \( \alpha_1 \), \( \alpha_2 \), and \( \alpha_3 \) are temperature dependent, being defined by the equations

\[
\alpha_1 = \frac{\varepsilon(T_s)A_{sr}}{C_p(T_s)m}; \quad \alpha_2 = \frac{h_{conv}(T_s, T_e)A_{se}}{C_p(T_s)m}; \quad \alpha_3 = \frac{\alpha_s A_{si} g_f}{C_p(T_s)m}.
\]  

(3)

The terms in (3) are described in Table I. The thermodynamic data used in this article (for SiC material) were obtained from [12]. It is assumed that the maximum value of the solar flux \( \max(G_s(t)) \) is 1000 W m\(^{-2}\) and that the furnace gain is \( g_f = 1000 \). Because the solar furnaces shown in Figure 3(a) and (b) have a value of \( g_f \) much higher than 1000, this value is selected as a lower bound on \( g_f \). Losses by thermal radiation occur all over the sample surface. Losses by convection from the upper and lower surfaces are combined in the convection factor

\[
h_{conv}(T_s, T_e) = 1.91((T_s - T_e)/L_e)^{0.25},
\]  

(4)

where laminar flow of air is assumed to occur [12].
In order to formulate the optimal control problem, the following cost function defined in the interval $[t_0, t_f]$ is considered

$$J(T(t), u(t)) = \frac{1}{2} f(R_T - T_s(t_f))^2 + \frac{1}{2} \int_{t_0}^{t_f} \left[ q(R_T - T_s(t))^2 + r(u(t) - u_0)^2 \right] dt,$$  

(5)

where the temperature reference $R_T$ is assumed to be constant in $[t_0, t_f]$: $f > 0$, $q > 0$, and $r > 0$ are adjustable parameters; and $u_0$ is defined as the nominal control value that generates the temperature reference value $R_T$. The aim is to use the parameter $r > 0$ in (5) to penalize offsets from $u_0$ without causing a large steady state tracking error.

The minimization of (5) is to be performed subject to the constraint imposed by the process model

$$\frac{d T_s(t)}{dt} = -\hat{\alpha}_1 [T_s^4(t) - T_e^4(t)] - \hat{\alpha}_2 [T_s(t) - T_e(t)] + \hat{\alpha}_3 u(t)$$  

(6)

that corresponds to (2), with $\hat{\alpha}_1$, $\hat{\alpha}_2$, and $\hat{\alpha}_3$ being constant estimates of $\alpha_1$, $\alpha_2$, and $\alpha_3$, and the manipulated variable is taken as $u(t) = G_s(t)s_f(u_s(t))$. In general, the simulation performed next assume that there is a mismatch between (6) and (2).

Using (6) and considering the steady state, it is concluded that the nominal control value $u_{R_T}$ that yields the equilibrium temperature $R_T$ under the constant environment temperature $T_e$ is given by

$$u_{R_T} = \frac{\hat{\alpha}_1}{\hat{\alpha}_3} [R_T^4 - T_e^4] + \frac{\hat{\alpha}_2}{\hat{\alpha}_3} [R_T - T_e].$$  

(7)

According to the optimal control methodology [13], the Hamiltonian function is given by

$$H(T_s(t), \lambda(t), u(t), t) = \frac{1}{2} \left[ q(R_T - T_s(t))^2 + r(u(t) - u_0)^2 \right]$$

$$+ \lambda(t) \left[ -\hat{\alpha}_1 (T_s^4(t) - T_e^4(t)) - \hat{\alpha}_2 (T_s(t) - T_e(t)) + \hat{\alpha}_3 u(t) \right].$$  

(8)

For an optimal trajectory for the state, $T^*_s$, and for the costate, $\lambda^*$, the minimization of $H(T_s(t), \lambda(t), u(t), t)$ with respect to $u(t)$ yields the optimal control signal\footnote{The symbols ($^*$) and ($_*$) are used to represent the optimal control and state or costate.} $u^* = h(T_s^*(t), \lambda^*(t), t)$. The optimal state verifies $(\dot{T}_s^*(t) = \left( \frac{\partial H}{\partial \lambda} \right)_*)$ or

$$\frac{d T_s^*(t)}{dt} = -\hat{\alpha}_1 [T_s^4(t) - T_e^4(t)] - \hat{\alpha}_2 [T_s^*(t) - T_e(t)] + \hat{\alpha}_3 u^*(t).$$  

(9)
with boundary value $T_s(t_0) = T_s0$. The optimal costate verifies $\left(\dot{\lambda}^*(t) = \left(\frac{\partial H}{\partial T_s}\right)_*, (t)\right)$.

$$\frac{d\lambda^*(t)}{dt} = \left(4\tilde{a}_1T_s^3(t) + \tilde{a}_2\right)\lambda(t) + q\left(R_T - T_s^*(t)\right),$$

with boundary value defined at $t_f$ by $\lambda^*(t_f) = \left(\frac{\partial H}{\partial T_s}\right)_*(t_f)$ or

$$\lambda^*(t_f) = -f(R_T - T_s^*(t_f)).$$

where $S = \frac{1}{2}f(R_T - T_s(t_f))^2$ is the term in the cost function that penalizes the deviation of $T_s(t_f)$ from the temperature reference at $t_f$. From (11) and if $T_s^*(t_f) = R_T$, it implies that $u^*(t_f) = u_{R_T}$ and $\lambda^*(t_f) = 0$, meaning that the control law must associate the condition $\lambda^*(t_f) = 0$ with $u^*(t) = u_{R_T}$ for a null steady tracking error.

3.1. Optimal control condition

In the present case, the control is constrained by a lower bound $u_{min} = 0$ and by an upper bound $u_{max}$ that depend on the available sun power that may change over time. Because of the constraints on $u(t)$, the principle of Pontryagin is used [14], meaning that $u^*(t)$ must be selected such that

$$H(T_s^*(t), \lambda^*(t), u^*(t), t) \leq H(T_s(t), \lambda^*(t), u(t), t).$$

Using (8) together with (12) yields the condition

$$\frac{1}{2}r(u^*(t) - u_0)^2 + \lambda^*(t)\tilde{a}_3u^*(t) \leq \frac{1}{2}r(u(t) - u_0)^2 + \lambda^*(t)\tilde{a}_3u(t).$$

In other words, the optimal control $u^*(t)$ must minimize the function $g(\lambda^*(t), u(t)) = \frac{1}{2}r(u(t) - u_0)^2 + \lambda^*(t)\tilde{a}_3u(t)$ subject to the constraint that $u(t) \in [0; u_{max}]$. The graphic of the function $g(\lambda^*(t), u(t))$ is depicted in Figure 4, where $\frac{1}{2}r(u(t) - u_0)^2$ and $\lambda^*(t)\tilde{a}_3u(t)$ are also shown. From this figure, it is concluded that if $\lambda^* > 0$, the minimum $u^* \in [0; u_0]$; if $\lambda^* = 0$, then $u^* = u_0$; and if $\lambda^* < 0$, the minimum $u^* \in [u_0; u_{max}]$.

Assuming that there are no constraints on $u$, the minimum $u^*$ is computed using $\partial g(\lambda^*, u)/\partial u = 0$, yielding $u^*(t) = u_0 - (\alpha_3/r)\lambda^*$.

With the constraint $u \in [0; u_{max}]$, the control function $u^*(t) = h(T_s^*(t), \lambda^*(t), t)$ is given by

$$u^*(t) = \begin{cases} 
  u_{max} & \text{if } \lambda^*(t) \leq -(u_{max} - u_0)\frac{r}{\alpha_3} \\
  u_0 - \frac{\tilde{a}_3}{r}\lambda^*(t) & \text{if } -(u_{max} - u_0)\frac{r}{\alpha_3} < \lambda^*(t) < u_0\frac{r}{\alpha_3} \\
  0 & \text{if } \lambda^*(t) \geq u_0\frac{r}{\alpha_3} 
\end{cases}.$$

![Figure 4. Example of a plot of the function $g(\lambda^*(t), u(t))$ with $u_0 = 3$, for $\lambda^*(t) > 0$ (on the left side) that has an absolute minimum in the interval $[0; u_0]$ and for $\lambda^*(t) < 0$ (on the right side) that has an absolute minimum in the interval $[u_0; u_{max}]$.](image-url)
3.2. Approximating the solution of the optimal control

A good numerical algorithm to approximate the solution of the optimal control problem results from the combination of an accurate integration method for the state and adjoint DEs, together with a good optimization algorithm [15]. The point to remark in the dynamics of solar furnaces is the fact that the expression for the derivative includes a fourth power of the temperature. To solve the problem of integrating the state and costate DEs, a Runge–Kutta method is used.

To approximate the optimal control solution, the following iterative algorithm is used:

1. Select an initial costate \( \lambda_{(i)}(t) \) with \( i = 0 \). A possible selection is \( \lambda_{(0)}(t) = 0 \) that yields \( u(t) = u_{RT} \).
2. Compute the control signal using (14) and represent it by \( u_{(i)}(t) \).
3. Solve the state equation (9) forward in time with initial state \( T_s(t_0) \) and with \( u_{(i)}(t) \) that was computed in the previous point. Let the solution at this step be represented by \( T_s(i)(t) \).
4. Solve the costate equation (10) backward in time using the terminal condition, \( \lambda(t_f) = -f(R_T - T_s(i)(t_f)) \) and the temperature reference \( R_T \). Increase the index \( i \) by one and represent the solution of the costate equation in this step by \( \lambda_{(i)}(t) \).
5. Jump to point 2 and repeat the iterative process until a suitable convergence criterion is met.

3.3. Analysis of the iterative method to approximate the optimal control solution

When using the iterative method described in 3.2 to compute an approximation to the optimal control solution, it is important that it converges to a solution that is near the optimal one. Because the analysis is complicated, a simplification is used to evaluate the convergence in the initial two steps of the algorithm, which depends on several parameters, \( T_s(t_0) \), \( \lambda(t_f) \), \( f \), \( r \), and \( q \), and also on the integration method. To address this problem, the equation satisfied by the tracking error \( e(t) = R_T - T_s(t) \) is used

\[
\frac{de(t)}{dt} = \hat{\alpha}_1 \left[ (R_T - e(t))^4 - T_e^4 \right] + \hat{\alpha}_2 (R_T - e(t) - T_e) - \hat{\alpha}_3 u(t). \tag{15}
\]

This equation is linearized assuming that the tracking error is much smaller than the temperature reference, that is, \((R_T - e(t))^4 \approx T_R^4 - 4T_R^3 e(t)\), yielding

\[
\frac{de(t)}{dt} = -\left( \hat{\alpha}_1 4T_R^3 + \hat{\alpha}_2 \right) e(t) + \hat{\alpha}_1 \left( T_R^4 - T_e^4 \right) + \hat{\alpha}_2 (R_T - T_e) - \hat{\alpha}_3 u(t). \tag{16}
\]

Using the same assumption \( (T_s(t) \approx R_T) \), the costate equation is approximated by

\[
\frac{d\lambda(t)}{dt} = + \left( \hat{\alpha}_1 4T_R^3 + \hat{\alpha}_2 \right) \lambda(t) + qe(t). \tag{17}
\]

Defining \( \theta := \hat{\alpha}_1 4T_R^3 + \hat{\alpha}_2 \) and considering that \( t \in [0; t_f] \) with \( t_0 = 0 \), the first (iterative) solution for the tracking error is given by

\[
e_{(0)}(t) = \exp^{(-\theta t)} e(0), \tag{18}
\]

where \( e(0) \) is the initial condition and represents the tracking error at \( t = 0 \). Note that with \( i = 0 \), \( \lambda_{(i)}(t) = 0 \) and the control signal \( u_{(0)} = u_{RT} \). The terminal condition of the costate equation is given by \( \lambda(t_f) = -f \exp^{(-\theta t_f)} e(0) \).

To solve the costate equation, a new variable is defined as \( \mu = t_f - t \) that represents the time reverse, and the costate is written in terms of \( \lambda_r(\mu) = \lambda(t_f - \mu) \) and \( e_r(\mu) = e(t_f - \mu) \), yielding

\[
\frac{d\lambda_r(\mu)}{d\mu} = -\theta \lambda_r(\mu) - qe_r(\mu), \tag{19}
\]

with initial condition \( \lambda_r(0) = -f \exp^{-\theta t_f} e(0) \). By integrating the equation, the solution of this IVP is given by

\[
\lambda_r(\mu) = \exp^{-\theta \mu} (-f) \exp^{-\theta t_f} e(0) - q \frac{1}{2\theta} \exp^{-\theta(t_f + \mu)} \left( \exp^{2\theta \mu} - 1 \right) e(0). \tag{20}
\]
that can be written in the forward time variable $t$ and gives the second iterative solution $\lambda_{(1)}(t)$,

$$
\lambda_{(1)}(t) = -f \exp^{-\theta(2t_f-t)} e(0) - \frac{q}{2\theta} \left( \exp^{-\theta t} - \exp^{-\theta(2t_f-t)} \right) e(0)
$$

(21)

the first solution $\lambda_{(0)}(t)$ being selected to be equal to 0.

Using (14) and assuming that there is no saturation, the control signal at this iterative step $i = 1$ is given by $u_i(t) = u_{RT} - \frac{\lambda_{(1)}(t)}{R}$. This is used in the dynamic equation of the tracking error to compute the tracking error $e_{(1)}(t)$ with $t \in [0; t_f]$. The result is

$$
e_{(1)}(t) = \exp^{-\theta t} e(0) \left\{ 1 - \frac{\hat{\alpha}_3^2}{2\theta} \frac{t_f}{\beta_r} \left[ \left( \frac{\beta_f}{t_f} - \frac{1}{2\theta t_f} \right) \left( \exp^{-2\theta(t_f-t)} - \exp^{-2\theta t_f} \right) + \frac{t}{t_f} \right] \right\}.
$$

(22)

Without loss of generality, one can choose $f = \beta_f/t_f$, $q = 1/t_f$ and $r = \beta_r/t_f$, with $\beta_f \geq 0$ and $\beta_r \geq 0$. This selection has two purposes: the first being to normalize the cost function such that it can be interpreted as the mean value of the square ‘error’, and the second is to reduce the number of parameters.

This result is used to relate the tracking error at step $i = 0$, $e_{(0)}(t)$ with the tracking error at step $i = 1$, $e_{(1)}(t)$, that is given by

$$
e_{(1)}(t) = e_{(0)}(t) \phi(t, t_f),
$$

(23)

with $\phi(t, t_f)$ defined by

$$
\phi(t, t_f) = \left\{ 1 - \frac{\hat{\alpha}_3^2}{2\theta} \frac{t_f}{\beta_r} \left[ \left( \frac{\beta_f}{t_f} - \frac{1}{2\theta t_f} \right) \left( \exp^{-2\theta(t_f-t)} - \exp^{-2\theta t_f} \right) + \frac{t}{t_f} \right] \right\}.
$$

(24)

In order to obtain monotonic convergence from the beginning of the iterative process, then $|e_{(i+1)}(t)| \leq |e_{(i)}(t)|$ and, in particular $|e_{(1)}(t)| \leq |e_{(0)}(t)|$, implying that $|\phi(t, t_f)| \leq 1$. If the parameters $r, f, q$, and $t_f$ are not carefully chosen, then the algorithm will increase the tracking error from the first iteration to the second iteration. For example, by choosing the ratio $\beta_r$ small enough and if $\beta_f$ is much larger than $1/(2\theta)$, then the tracking error at the second iteration for $t = t_f$ is given by

$$
e_{(1)}(t_f) = e_{(0)}(t_f) \left\{ 1 - \frac{\hat{\alpha}_3^2}{2\theta} \frac{1}{\beta_r} \left[ \left( \frac{\beta_f}{t_f} - \frac{1}{2\theta} \right) + 1 \right] \right\}
$$

(25)

and $|\phi(t, t_f)|$ can be made larger than 1. This result is used to guide the selection of the parameters to solve the problem numerically.

3.4. Offline approximation of the optimal control solution

From the discussion presented in Section 3.3, it is clear that it is very difficult to obtain the exact solutions of the state and of the costate for the iteration index $i \geq 2$. To obtain additional information, numerical computer results are used to study the algorithm convergence.

Without loss of generality, the temperature reference is selected as $R_T = 1000$ K, and the initial value of the temperature is $T_s(0) = 400$ K. In fact, with this selection, the tracking error is not ‘much smaller’ if compared with the temperature reference $R_T$ as assumed in Section 3.3. The values of the model terms are selected equal to $\hat{\alpha}_1 = 6.456 \times 10^{-12}$, $\hat{\alpha}_2 = 3.5 \times 10^{-3}$, $\hat{\alpha}_3 = 0.1059$, and $q = 1/t_f$. The other parameters that enter in the cost function $f = \beta_f/t_f$, $r = \beta_r/t_f$, and $t_f$ are changed to evaluate the convergence of the algorithm. Figures 5, 6, and 7 show the iterative solutions of the costate, control, and state for $t_f = 50s$, $\beta_f = 1.0$, and $\beta_r = 10.0$.

Figures 8, 9, and 10 show the iterative solutions of the costate, control, and state but now $t_f = 25$ s, with the other parameters having the same values as in the previous computer simulation. The main conclusion that can be drawn is that the convergence is much faster because the solutions for $i = 5, 8, 11, 14$ are quite closed.

The next set of computer simulations is used to compute the optimal value of the cost function as a function of $t_f$ and of the weight on control, $\beta_r$, that can be used to adjust the speed of the
response of the control system. The results are shown in Figure 11. The cost function increases with increasing values of the weight on control $\beta_r$ for constant $t_f$. The cost function decreases with increasing values of the prediction horizon $t_f$ for constant $\beta_r$. From these conclusions, a logic option
is to select a high value for the prediction horizon \( t_f \) and a low value for the control weight \( \beta_r \) to obtain a faster convergence to the reference. However, the algorithm is not able to converge for large values of the prediction horizon \( t_f \) and small values of the weight on control \( \beta_r \), corresponding to
the region represented in the Figure 11 by a blue flat surface. A practical approach is to use a small value for the prediction horizon and a small value for the control weight. These conclusions about the convergence of the algorithm are in line with the conclusions that were drawn from the simplified analysis presented in Section 3.3. Other algorithms to solve the state and costate equations could be explored to overcome the convergence problem that occurs with the algorithm described.

3.5. Receding horizon algorithm

In order apply the optimal control algorithm in a real-time environment, the algorithm described in Section 3.2 is used at each discrete time $k$. The common approach, as described in [14], is to iterate the algorithm until convergence is reached. Here, we take advantage of the receding horizon implementation and distribute these iterations on time, according to the following steps:

1. The temperature of the process is sampled, and the temperature reference is assumed to be constant along the prediction horizon ($t_f$).
2. A model of the temperature process is used to solve the state equation and the costate equation. The algorithm is iterated during a fix maximum number of iterations. The maximum number of iterations is constrained by the sampling time $h$, the time spent in each iteration, and available computer processing power.
3. The control value obtained for $u(t)$ is converted to $u_s(t)$ using $u_s(t) = s_f^{-1}(u(t))/G_s(t)$ that defines the aperture of the shutter and is applied to the plant.

4. SIMULATION RESULTS

To apply the control algorithm presented in the previous section, the values of seven parameters must be selected, namely $\hat{\alpha}_1$, $\hat{\alpha}_3$, $h$, $t_f$, $f$, $q$, and $r$, and it is assumed that $\hat{\alpha}_2 = 0$. The parameters that approximate the dynamics of the process were selected for a temperature of 1000 K, being $\hat{\alpha}_1 = 6.456 \times 10^{-12}$ and $\hat{\alpha}_3 = 0.059$. Note that $\hat{\alpha}_2 = 0$ to force a mismatch between the model used in the controller and the one employed to simulate the plant and to introduce a static error. The sampling time is $h = 0.5 \text{ s}$. The temperature reference was selected to evolve by successive steps and ramps, between 300 and 1300 K. Note that $\alpha_1$, $\alpha_2$, and $\alpha_3$ change with temperature. As an example, for $T_s = 300 \text{ K}$, they are, respectively, $\{1.16 \times 10^{-11}$, $1.8 \times 10^{-3}$, $1.4 \times 10^{-1}\}$, and for $T_s = 1000 \text{ K}$, they are, respectively, $\{6.4 \times 10^{-12}$, $3.5 \times 10^{-3}$, $1.06 \times 10^{-1}\}$. The variability of the available sun power is simulated according to the signal shown in Figure 12, changing from 400 to 900 W m$^{-2}$. It simulates the power fluctuations caused by moving clouds.
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Figure 12. Simulation of the variability of the available sun power that can be caused by moving clouds.

Figure 13. Simulation results obtained with the optimal controller where the parameter $r$ was selected as a large value such that $u(t) = u_{RF}$. The power signal $u(t)$ is converted to the aperture of such $u_s(t)$ using a strategy to compensate the variability of the power of the sun.

To evaluate the application of the optimal controller with receding horizon and to establish a figure of reference to compare the results, the parameter $r$ is selected as a very large value such that the process is driven only by $u_{RF}$. The results obtained with this controller configuration are presented Figure 13(a) and (b). It is clear that the nominal control action $u_{RF}$ that is computed using (7) is not adequate because there is a large tracking error.

The results shown in Figure 14(a) and (b) are obtained with the prediction time $t_f = 5 \text{ s}$, $f = 0.001/t_f$, $q = 1/t_f$, and $r = 0.2/t_f$. The value of the parameter $r$ was decreased to obtain a smaller tracking error and to obtain a smooth control signal with ‘low’ noise. The full range of the control signal $u_s(t)$ is used. The temperature of the sample does not have overshoot, which is an important practical aspect to avoid melting the sample.

The tracking error can be decreased by decreasing the parameter $\beta_f$. The results that are shown in Figure 15(a) and (b) were obtained with $t_f = 2 \text{ s}$, $f = 0.001/t_f$, $q = 1/t_f$, and $r = 0.05/t_f$, and the parameter $r$ was reduced approximately by half of the value of the previous simulation. The tracking error is decreased, but there is an increase of the noise in the control signal that may not be acceptable for the long-term operation of the shutter.
Figure 14. Controller with parameters \( t_f = 5 \text{ s}, f = 0.001/t_f, q = 1/t_f, \) and \( r = 0.2/t_f \). The value of parameter \( r \) was decreased to obtain a small tracking error.

Figure 15. Controller with parameters \( t_f = 2 \text{ s}, f = 0.001/t_f, q = 1/t_f, \) and \( r = 0.05/t_f \). By decreasing the value of the parameter \( r \), there is a decrease of the tracking error, but there is an increase of the variability of the control signal that may not be acceptable for the long-term operation of the shutter.

5. CONCLUSION

This work explores the application of the optimal control methodology, combined with a receding horizon approach, to the control of temperature on a solar furnace. This process is characterized by having nonlinear effects that may pose problems during stress material tests. The control problem is formulated in continuous time domain.

Because the plant nonlinearity is a fourth-order power, it is not convenient to use a Euler method. Instead, model discretization has been performed by numerical integration of the DE with a Runge–Kutta method, both for the state and costate equations. The iterative algorithm applied to solve the optimal control problem was analyzed. It is concluded that the cost function increases with increasing values of the weight on control \( \beta_r \) for constant \( t_f \), and the cost function decreases with increasing values of the prediction horizon \( t_f \) for constant \( \beta_r \). From these observations, a logic option is to select a high value for the prediction horizon \( t_f \) and a low value for the control weight \( \beta_r \) to obtain a faster tracking. The algorithm is not however able to converge when large values of
prediction horizon $t_f$ and small values of the weight on control $\beta_r$ are selected. A practical approach is to use a small value for the prediction horizon and a small value for the control weight; with this option, a compromise is obtained between convergence of the algorithm and a low value of the cost function. Computer simulations are used to assess the performance of the controller either as offline simulation or in a simulated real-time environment. These simulations show that the controller can be tuned by adjusting the weight on control $\beta_r$ and the prediction horizon $t_f$ to achieve a compromise between the main objective of reference tracking and an acceptable variability for the shutter position. However, the iterative algorithm used to solve the optimal control problem at each sampling time $t = t_0 + k \Delta t$ imposes a larger computational burden on the controller computer. The practical implementation depends on the computational power of the computer. An approach that can be used to reduce computational burden is to formulate the optimal control problem in the discrete time framework, where the sampling time can be used as a tuning knob to balance the control performance with a reduction of the computational burden of the algorithm.

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