
$1^{st}$ Progress Report

OPERATIONAL AMPLIFIER BASED TRANSIMPEDEANCE AMPLIFIERS

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Abstract

Low noise transimpedance amplifiers (TIAs) are used in the front-end of radiation detectors to convert the current pulses generated by a light sensitive device into voltage pulses with specified amplitude and shape.

A review at block diagram level is presented of the feedback TIA, which uses one operational amplifier (OA). Pulse shape analysis is performed considering the sensor capacitance and assuming that the OA has a first order gain function. Equations are derived for the TIA transimpedance function and rms output noise voltage.

Numerical results are obtained considering the specifications of a PET (Positron Emission Tomography) system for high resolution mammography. We conclude that for high speed and low noise applications, the feedback capacitances needed are very small, of the order of the parasitic capacitances, and the gain-bandwidth product of the OA should be very high. This means that for this application a two-stage TIA should be used.
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1. INTRODUCTION

This report is the first progress report on the research of the first author with a view to obtaining a PhD degree under the supervision of the second author. The work was performed at the Analog and Mixed Signals Group of INESC-ID, Lisbon, within the framework of project TARDE (PTDC/EEA-ELC/65710/2006), funded by FCT.

The motivation for this work was originated by the participation of the Analog and Mixed Signals Group in Project PET (Positron Emission Tomography for Mammography [1,2]).

A review of the transimpedance amplifier is presented as the input amplifier in the signal processing chain for radiation detection. The specifications of the transimpedance amplifier considered in the study reported here are those of project PET, which are difficult to fulfil, since a high gain and low noise with a high bandwidth are required to achieve the desired output voltage amplitude and peaking time.

This report presents a review, from a systems point of view (at block diagram level), of a single stage transimpedance amplifier, which leads to the conclusion that it cannot satisfy the specifications. It was found that the capacitances involved are of the order of the parasitic capacitances, and the gain-bandwidth product is beyond the technology limits, with values of the order of tens of GHz.

In order to get a clear insight into the relations between the parameters of the amplifier, a step-by-step methodology was followed. Thus, the simplest case is first treated in Section 3, where an ideal operational amplifier (OA) is considered, in order to have a first, yet rough, idea of the circuit parameters required.

More realistic situations are treated in Sections 4 and 5. In Section 4, the OA is approximated by a first order system with a dominant pole. In Section 5 the effect of the capacitance at the amplifier input is considered.

A stability study is presented in Section 6, and noise analysis is presented in Section 7. Section 8 presents a preliminary study of a 2-stage architecture with ideal OAs.
2. THE INPUT AMPLIFIER IN RADIATION DETECTORS

In the signal processing chain of radiation detectors, the detecting device, which is assumed to be an avalanche photo-diode (APD), is attached to an input amplifier (charge sensitive or transimpedance amplifier) as shown in Fig.1(a). The APD is reverse biased through the resistance $R$ and the high voltage bias is blocked from the amplifier by the coupling capacitor $C$.

![Fig. 1 – APD connection to the amplifier: (a) APD bias; (b) Equivalent circuit](image)

The equivalent circuit of the APD is a current source with current $i_d$ in parallel with a capacitance $C_d$, as shown in Fig.1(b). In response to a light pulse, the current source produces a pulse $i_d$ with a sharp rise and an exponential decay. The input amplifier receives this current pulse and produces a voltage pulse with the required shape and amplitude.

In this report the values from Project PET [1,2] will be extensively used in the examples, and as guidelines to the approximations. The APD chosen has a rise time of 2-3 ns and an exponential decay with a time constant of about 40 ns, as shown in Fig.2. For approximate calculations the rise time is neglected, and it is assumed that the current pulse $i_d(t)$ in Fig. 2 is exponential:

\[ i_d = I_{dm} e^{-\frac{t}{\tau_d}} \quad \text{for} \quad t \geq 0 \quad (1) \]

The Laplace transform is

\[ I_d(s) = \frac{I_{dm} \tau_d}{1 + s \tau_d} \quad (2) \]

The peak value of the current pulse, $I_{dm}$, the time constant, $\tau_d$, and the APD capacitance, $C_d$, are [1,2]:

---
\[ I_{dm} = 2.25 \, \mu A \quad \tau_d \approx 40 \, ns \quad C_d \approx 10 \, pF \]

The specifications of the input amplifier are that the maximum peak value of the output voltage \( V_{om} \) should be 1 V and the peaking time \( t_m \) should not exceed 40 ns.

![Diagram showing the current and voltage pulses](image)

*Fig. 2 – Input amplifier pulse shaping: APD current and amplifier output voltage.*

The signal processing of the PET system requires that the amplifier output signal be a voltage pulse \( v_o \) with the shape shown in Fig. 2. This means that the amplifier should perform a pulse shaping function, in addition to signal amplification [1,2,3].
In Fig. 3 we consider that the operational amplifier (OA) is ideal (i.e, the voltage gain $A \rightarrow \infty$). This means that the amplifier input voltage is zero and, thus, we can neglect the APD capacitance, $C_d$. With an exponential current pulse, defined by (1), the output voltage is:

$$V_o(s) = -\frac{R_f}{1+s\tau_f} I_d(s) \quad \text{with} \quad \tau_f = R_f C_f$$  \hspace{1cm} (3)

Replacing (2) in (3)

$$-\frac{V_o}{R_f I_{dn}} = \frac{\tau_d}{(1+s\tau_f)(1+s\tau_d)}$$  \hspace{1cm} (4)

For $\tau_f \neq \tau_d$ (4) has two real poles at $s = -1/\tau_f$ and $s = -1/\tau_d$. The corresponding equation in the time domain is:

$$-\frac{v_0}{R_f I_{dn}} = \frac{\tau_d}{(\tau_f - \tau_d)} \left[ e^{-t/\tau_f} - e^{-t/\tau_d} \right] \quad \text{if} \quad \frac{\tau_f}{\tau_d} \neq 1$$  \hspace{1cm} (5)

If $\tau_f = \tau_d$, the network function in (4) has a double pole at $s = -1/\tau_d$, and the time-domain equation is:
\[ -\frac{v_o}{R_f I_{dm}} = \frac{I}{\tau_d} e^{-\frac{t}{\tau_d}} \]  

(6)

Fig. 4 shows the response to the exponential pulse for different values of \( \tau_f / \tau_d \).

![1-STAGE TRANSIMPEDEANCE AMPLIFIER WITH IDEAL OA](image)

**Fig. 4** – Response of the transimpedance amplifier with one ideal OA to the exponential pulse for different values of \( \tau_f / \tau_d \).

As an example, we consider \( \tau_f / \tau_d = 0.5 \) and, from Fig. 4, we obtain \( t_m = 28 \text{ ns} \). To obtain an output voltage peak value \( V_{om} = 1 \text{ V} \), it is required \( R_f \approx 888.9 \text{ k}\Omega \), and therefore \( C_f = 22.5 \text{ fF} \).

These results were verified by a SPICE simulation using the circuit in Fig. 5, where I1 corresponds to the exponential current source and E1 is a voltage controlled voltage source with the gain set to \( 10^5 \), which models the ideal OA. The response obtained from SPICE simulation is presented in Fig. 6 and shows that

\[ V_{om} = 1 \text{ V} \text{ and } t_m = 28 \text{ ns} \]

which agree with the theoretical results.
Fig. 5 – Circuit used in the SPICE simulation of the transimpedance amplifier with one ideal OA for the case $\tau_f/\tau_d = 0.5$

Fig. 6 – SPICE simulation of the amplifier response with $\tau_f/\tau_d = 0.5$.

The main conclusion from this first ideal approximation is that the capacitances $C_f$ involved are of the order of tens of fF, which is the order of the parasitic capacitances.
4. ONE-STAGE TRANSIMPEDANCE AMPLIFIER USING ONE OA WITH DOMINANT POLE

Real OA’s have finite gain and their transfer function is often approximated by considering that there is a dominant pole

\[ A = \frac{A_o}{1 + s \tau_a} \]  \hspace{1cm} (7)

With this approximation, the OA is a first order system with cut-off frequency \( \omega_a = 1/\tau_a \), and a constant gain \( A_o \gg 1 \) at low frequencies. The shape of the amplitude Bode plot is represented in Fig. 7.

\[ A = \frac{A_o}{1 + s \tau_a} \]

If the frequencies \( \omega_i \) of the input signal are such that \( \omega_i >> \omega_a \), then in (7) \( |s \tau_a| >> 1 \), meaning that the amplifier’s transfer function can be approximated by the linear part of the Bode plot

\[ A \approx \frac{A_o}{s \tau_a} = \frac{B}{s}, \quad B = \frac{A_o}{\tau_a} \]  \hspace{1cm} (8)

where \( B \) is the gain-bandwidth product of the amplifier.

Neglecting the APD capacitance \( C_d \), the normalized response is:

Fig. 7 –Bode plot of an OA with dominant pole.
\[
\frac{V_o}{R_f I_{dm}} = \frac{\tau_d}{(1 + \frac{1}{A})(1 + s \tau_f)(1 + s \tau_d)}
\]  
(9)

Taking into consideration (8)

\[
\frac{V_o}{R_f I_{dm}} = \frac{B \tau_d}{(1 + s B^{-1})(1 + s \tau_f)(1 + s \tau_d)}
\]  
(10)

This transfer function has three poles at \(s = -B\), \(s = -1/\tau_f\) and \(s = -1/\tau_d\). The corresponding equations for \(v_o(t)\) are presented in Table 1 for different cases concerning the parameters \(\tau_f, \tau_d\) and \(B\).

As an example, we consider the case where \(B \tau_d = 10\) and. Fig. 8 shows the response to the exponential pulse for this case considering different values of \(\tau_f/\tau_d\). If \(\tau_f/\tau_d = 0.5\), to obtain \(V_{om} = 1\) V we must have:

\[
R_f = \frac{1}{0.494 \times 2.25 \times 10^{-6}} \approx 900 \text{ k\(\Omega\)}
\]

\[
C_f = \frac{0.5 \times 4 \times 10^{-8}}{899.87 \times 10^5} = 22.2 \text{ f\(\text{F}\)}
\]

and the peaking time is

\[
t_m \approx 0.4 \times 40 \text{ ns} = 32.4 \text{ ns}
\]

If \(A_o = 10^3\), to have \(B \tau_d = 10\) we need

\[
B \approx 39.8 \text{ MHz}
\]

\[
\tau_a = 4 \mu\text{s}
\]

These results were verified by a SPICE simulation using the circuit represented in Fig. 9. The block LAPACE1 corresponds to the amplifier with its transfer function specified and I1 corresponds to the exponential current source.

Simulation results are presented in Fig. 10, which show that \(V_{om} = 1\) V and \(t_m = 32.2\) ns, which agree with the theoretical results.
Table 1 – Response of the transimpedance amplifier using one OA with dominant pole (APD capacitance neglected)

<table>
<thead>
<tr>
<th>Frequency Domain</th>
<th>Condition</th>
<th>Time Domain</th>
</tr>
</thead>
<tbody>
<tr>
<td>[- \frac{V_o}{R_f I_{dm}} = \frac{B \tau_d}{(B + s)(1 + s \tau_f)(1 + s \tau_d)}]</td>
<td>(\tau_f \neq \tau_d \neq \frac{1}{B})</td>
<td>[- \frac{V_o}{R_f I_{dm}} = B \tau_d \left[ \frac{\tau_f e^{-t/\tau_f}}{(\tau_f - \tau_d)(B \tau_f - 1)} + \frac{e^{-Bt}}{(B \tau_d - 1)(B \tau_f - 1)} - \frac{\tau_d e^{-t/\tau_d}}{(\tau_f - \tau_d)(B \tau_d - 1)} \right]]</td>
</tr>
<tr>
<td>(\tau_f = \frac{1}{B} \neq \tau_d)</td>
<td>[- \frac{V_o}{R_f I_{dm}} = \frac{\tau_d^2}{(\tau_f - \tau_d)^2} \left[ e^{-t/\tau_d} + \frac{1}{(\tau_d - \tau_f)}(t-1) e^{-t/\tau_f} \right]]</td>
<td></td>
</tr>
<tr>
<td>(\tau_f \neq \tau_d = \frac{1}{B})</td>
<td>[- \frac{V_o}{R_f I_{dm}} = \frac{\tau_d}{(\tau_f - \tau_d)^2} \left[ \tau_f e^{-t/\tau_f} + \frac{1}{(\tau_d - \tau_f)}(t-1) e^{-t/\tau_d} \right]]</td>
<td></td>
</tr>
<tr>
<td>(\tau_f = \tau_d \neq \frac{1}{B})</td>
<td>[- \frac{V_o}{R_f I_{dm}} = B \tau_d \left[ e^{-Bt} + \frac{(B - 1)(t-1)}{(B \tau_d - 1)} e^{-t/\tau_d} \right]]</td>
<td></td>
</tr>
<tr>
<td>(\tau_f = \tau_d = \frac{1}{B})</td>
<td>[- \frac{V_o}{R_f I_{dm}} = \frac{1}{2} \frac{t^2}{\tau_d^2} e^{-t/\tau_d}]</td>
<td></td>
</tr>
</tbody>
</table>
Fig. 8 - Response of the transimpedance amplifier with dominant pole ($B\tau_d = 10$).

Fig. 9 – Circuit used in the SPICE simulation of the transimpedance amplifier with dominant pole OA ($B\tau_d = 10$).
Fig. 10 – SPICE simulation of the amplifier response to the exponential pulse with $B\tau_d = 10$ and $\tau_f/\tau_d = 0.5$

Further simulations have shown that fast responses of the amplifier are obtained when $\tau_f \approx \tau_a$, meaning that the gain at frequency $\omega_f$ is $A_o\omega_f/\omega_f > 0$ dB (see Fig. 9), which means that

$$R_j C_j >> \frac{\tau_a}{A_o}$$

(11)

This condition will be considered throughout this report.
5. EFFECT OF INPUT SOURCE CAPACITANCE

In the transimpedance amplifier if the OA is considered ideal, as we did in Section 2, the source capacitance $C_d$ has no effect on the network function of the circuit. If the OA is not ideal this assumption is not applicable. In that case an input voltage $-V_o/A(s)$ should be considered (see Fig.12), with $A(s)$ defined by (7).

For the circuit in Fig.11, the output voltage $V_o$ is:

$$-V_o = \frac{R_f I_d}{1 + \frac{1}{A(s)} + sR_f C_f + s \frac{R_f}{A(s)} (C_f + C_d)}$$

Replacing $A(s)$ by (7) it results:

$$-V_o = \frac{R_f I_d}{1 + s \left[ R_f C_f + \frac{\tau_a}{A_0} + R_f \frac{C_d}{A_0} \right] + s^2 R_f \left( C_f + C_d \right) \frac{\tau_a}{A_0}}$$

It should be noted that the results of the previous section can be obtained from (13) by making $C_d = 0$.

Thus, assuming in (13) that

$$C_d \gg C_f$$

(14)
and considering that condition (11) applies,

\[ R_j C_f \gg \frac{\tau_a}{A_o} \]

is it results:

\[ -V_o = \frac{R_j I_d}{1 + sR_j \left( C_f + \frac{C_d}{A_o} \right) + s^2 R_j C_d \frac{\tau_a}{A_o}} \quad (15) \]

Condition (14) applies to radiation detectors in which the source is an APD [1,2,3]. With the APD used in project PET [1,2] \( C_d \) is about 10 pF, and \( C_f \) is of the order of hundreds of fF. It can also be shown that if (14) is verified and if the poles in (15) are real, (11) is verified for any practical situation.

We now assume that the poles in (15) are real with time constants \( \tau_1 \) and \( \tau_2 \),

\[ -V_o = \frac{R_j I_d}{(1 + s\tau_1)(1 + s\tau_2)} \quad (16) \]

and that the approximation \( \tau_1 \gg \tau_2 \) is valid, which allows a simplified mathematical treatment. This is not, however, the case in project PET, where \( t_m \) is a short peaking time of the order of \( \tau_d \). In this case \( \tau_1 \approx \tau_2 \ll \tau_d \) produces better results, making better use of the available gain-bandwidth product. Fig.12 shows the response of the amplifier with \( \tau_1 = \tau_2 \) for several cases of \( \tau_2 / \tau_d \).

Although the assumption \( \tau_1 \gg \tau_2 \) may not be applicable, we will use it to simplify the equations. Comparison of (16) with (15) leads to

\[ \tau_1 \approx R_j \left( C_f + \frac{C_d}{A_o} \right) \quad (17) \]

\[ \tau_2 = \frac{C_d}{C_d + A_o C_f} \tau_a \quad (18) \]

Making the substitution \( I_d = \frac{I_{dm}}{1 + s\tau_d} \) in (16) we obtain

\[ -\frac{V_o}{R_j I_{dm}} = \frac{\tau_d}{(1 + s\tau_1)(1 + s\tau_2)(1 + s\tau_d)} \quad (19) \]

Table 2 presents the corresponding time-domain equations.
ONE STAGE AMPLIFIER WITH SOURCE CAPACITANCE $C_d$: $\tau_1/\tau_2 = 1$

Fig. 12 - Response of the transimpedance amplifier with one pole and source capacitance ($\tau_1/\tau_2 = 1$).

Fig. 13 shows the response of the amplifier with $\tau_1/\tau_2 = 5$ for several cases of $\tau_2/\tau_d$.

Considering $I_{dm} = 2.25 \mu$A, $\tau_d = 40$ ns, and $\tau_2/\tau_d = 0.1$, to obtain $V_{om} = 1$ V it is required $R_f = 900$ k$\Omega$ and the peaking time is $t_m = 32.8$ ns.

Considering $A_0 = 10^3$, from (19) and (20) we obtain

$$C_f \approx 12.23 \text{ fF} \quad \tau_a \approx 8.9 \text{ ns} \quad B = 17.9 \text{ GHz}$$

A small value of $C_f$ is obtained, which is of the order of the parasitic capacitances. The gain-bandwidth product is well above the value allowed by the technology selected in project PET, which is about 10 GHz [1,2].
ONE STAGE AMPLIFIER WITH SOURCE CAPACITANCE $C_d$: $\tau_1/\tau_2 = 5$

**Fig. 13** – Response of the transimpedance amplifier with one pole and source capacitance ($\tau_1/\tau_2 = 5$).

These results were verified by SPICE simulation. The circuit used is presented in Fig. 14. The block LAPLACE2 is, as before, an amplifier defined by its transfer function.

Results of simulation are shown in Fig. 15, from which we obtain $V_{om} \approx 1.1 \text{ V}$ and $t_m \approx 30.3 \text{ ns}$ when $\tau_1/\tau_2 = 5$.

**Fig. 14** – Circuit for SPICE simulation of the transimpedance amplifier with 1-pole OA and source capacitance $C_d$, for $\tau_1/\tau_2 = 1$ and $\tau_1/\tau_2 = 5$ ($\tau_1/\tau_d = 0.1$).
Table 2 – Response of the transimpedance amplifier to the pulse current considering the source capacitance $C_d$

<table>
<thead>
<tr>
<th>Frequency Domain</th>
<th>Condition</th>
<th>Time Domain</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_1 \neq \tau_2$</td>
<td>$\tau_1 \neq \tau_d$</td>
<td>$\tau_2 \neq \tau_d$</td>
</tr>
<tr>
<td>$\tau_1 \neq \tau_2$</td>
<td>$\tau_1 \neq \tau_d$</td>
<td>$\tau_2 = \tau_d$</td>
</tr>
<tr>
<td>$\tau_1 \neq \tau_2$</td>
<td>$\tau_1 = \tau_d$</td>
<td>$\tau_2 \neq \tau_d$</td>
</tr>
<tr>
<td>$\tau_1 = \tau_2 = \tau$</td>
<td>$\tau_1 \neq \tau_d$</td>
<td>$\tau_2 \neq \tau_d$</td>
</tr>
<tr>
<td>$\tau_1 = \tau_2 = \tau_d$</td>
<td>$-\frac{V_v}{I_{dm} R_1 R_2 R_3} = \frac{1}{2} \left( \frac{I_d}{\tau_d} \right)^2 e^{-\frac{t}{\tau_d}}$</td>
<td></td>
</tr>
</tbody>
</table>
Fig. 15 – SPICE simulation of the response of the amplifier with $\tau_1/\tau_2 = 1$ and $\tau_1/\tau_2 = 5$ ($\tau_2/\tau_d = 0.1$).

The difference between theoretical and SPICE results is due to the approximation considered, introducing errors of about 10% in the values of the capacitance for the case $\tau_1/\tau_2 = 5$.

As a concluding remark we can say that in order to meet the specifications of project PET the approximation $\tau_1 \gg \tau_2$ leads to:

i) $C_f$ of the order of few tens of fF, which is the order of the parasitic capacitances.

ii) gain-bandwidth product $B$ of tens of GHz, which is beyond the limit of about 10 GHz for the technology chosen (0.35µm AMS technology [1]).

Therefore, the use of this approximation should be carefully considered whenever very fast response times are needed.

Simulations performed for other possible combinations of $\tau_1$ and $\tau_2$ for the given $\tau_d$ have shown that it is impossible to meet the PET requirements with only one stage. The best cases require capacitance values of tens of fF and gain-bandwidth products near the technology limit and are achieved when $\tau_1 = \tau_2 \ll \tau_d$. The mathematical treatment of this case involves the use of the exact equations for $\tau_1$ and $\tau_2$, which are complicated and will not be considered here.
6. STABILITY

The network stability is determined by the analysis of the phase margin of the loop gain function $A(s)\beta(s)$ [4], where $\beta(s)$ is the feedback factor.

The feedback factor of the circuit in Fig. 11 is:

$$\beta(s) = \frac{1+s\tau_f}{1+s\tau_z}$$  \hspace{1cm} (20)

where

$$\tau_f = R_f \ C_f, \ \tau_z = R_f \ C_d$$  \hspace{1cm} (21)

The loop-gain is:

$$A(s)\beta(s) = \frac{A_0 (1+s\tau_f)}{(1+s\tau_d)(1+s\tau_z)}$$  \hspace{1cm} (22)

Assuming, as before, that $C_d >> C_f$, from (21) it follows that

$$\tau_z >> \tau_f$$  \hspace{1cm} (23)

The loop-gain function is of second order with one zero and two poles, which we assume to be real. Thus, this system is always stable. However, the stability margin depends on the relative position of $\tau_f$ and $\tau_z$.

If $\tau_f >> \tau_z$, the phase characteristic is always above -90°. Simulations have shown that when $\tau_1 >> \tau_2$ this condition is verified.

If $\tau_f \approx \tau_z$, the loop-gain can be approximated by a first order system with a pole at $\tau_z$. The phase margin is always higher than 90°.

If $\tau_f << \tau_z$ the phase characteristic is well above -180° if $\tau_f$ is less than $\tau_z$ by no more than a decade. For values higher than this limit, the phase is above -180° by a difference of few degrees, which corresponds to a low degree of stability.

Using values from project PET, calculations have shown that these two last conditions, $\tau_f \approx \tau_z$ and $\tau_f << \tau_z$, correspond to cases where a good trade off exists between the values of the capacitance $C_f$ and the gain-bandwidth product in order to meet the amplifier specifications.

As an example, we consider the results from the example presented in Section 6, where $\tau_1/\tau_2 = 5$ and $\tau_2/\tau_0 = 0.1$: 
\( A_0 = 10^3, \quad R_f \approx 900 \, \text{k\Ohm}, \quad C_f = 12.2 \, \text{fF}, \quad \tau_a = 10.9 \, \text{ns}, \quad \tau_f = 21.7 \, \text{ns}, \quad \tau_c = 0.1 \, \mu\text{s}. \)

Fig. 16 shows the Bode plot for this example. It can be seen that the phase will be always above \(-90^\circ\).

![Bode Diagram](image)

**Fig. 16** – Bode diagram of the loop-gain of transimpedance amplifier for \( \tau_1/\tau_2 = 5 \) and \( \tau_2/\tau_d = 0.1 \).

Spice simulations were carried out using the circuit of Fig. 17 to verify the results obtained.

![Circuit Diagram](image)

**Fig. 17** – Circuit used in the SPICE simulation of the loop-gain
The block LAPACE2 corresponds to the amplifier defined by its transfer function. R1 corresponds to $R_f$, C1 to $C_f$, and C2 to $C_d$, in Fig. 11. Fig. 18 shows the results obtained from the SPICE simulation, which agrees with the theoretical results obtained before.

Fig. 18 – Bode diagram obtained by SPICE simulation of the circuit in Fig. 17
7. NOISE ANALYSIS

Fig. 19 shows a one-stage transimpedance amplifier with an input equivalent noise source \( V_{in} \), in which \( V_{on} \) is the noise voltage at the output. If the input capacitance of the amplifier is neglected we have:

\[
sC_d \left( \frac{V_{in} + V_{on}}{A(s)} \right) = - \left( V_{on} + \frac{V_{on}}{A(s)} + V_{in} \right) \left( \frac{1}{R_f} + sC_f \right)
\]

or

\[
\frac{V_{on}}{V_{in}} = \frac{1 + sR_f (C_f + C_d)}{1 + \frac{1}{A(s)} + sR_f (C_f + C_d) A(s)}
\]

Assuming, as before, that the amplifier has a dominant pole

\[
A = \frac{A_0}{1 + s \tau_a}
\]

and also considering that \( R_f C_f >> \tau_a / A_0 \) and \( C_d >> C_f \)

\[
\frac{V_{on}}{V_{in}} = \frac{1 + sR_f C_d}{1 + sR_f (C_f + \frac{C_d}{A_0}) + s^2 (R_f C_d \frac{\tau_a}{A_0})}
\]

This is a 2\(^{nd}\) order transfer function with one zero and two poles,
\[ \frac{V_{on}}{V_{in}} = \frac{1 + s \tau_z}{(1 + s \tau_1)(1 + s \tau_2)} \]  

(27)

with

\[ \tau_z = R_i C_d \]  

(28)

By making \( s = j \omega \) in (27), with \( \omega_i = 2\pi f_i \)

\[ |A_i|^2 = \left| \frac{V_{on}}{V_{in}} \right|^2 = \frac{1 + (\omega/\omega_i)^2}{\left[ 1 + (\omega/\omega_i)^2 \right]^2} = \frac{1 + (f/f_i)^2}{\left[ 1 + (f/f_i)^2 \right]^2} \]  

(29)

Considering a system where \( X^2 \) is the power spectral density of a random input signal \( x(t) \), and \( Y^2 \) is the power spectral density of the response \( y(t) \) if the system has a transfer function \( A(s) \), then

\[ Y^2 = |A(j\omega)|^2 X^2 \]  

(30)

In the circuit of Fig. 19

\[ \overline{V_{on}} = |A_i|^2 \overline{v_{in}^2} \]  

(31)

and by using (29),

\[ V_{onrms}^2 = \int_0^\infty \frac{1 + (f/f_i)^2}{\left[ 1 + (f/f_i)^2 \right]^2} \overline{v_{in}^2} \, df \]  

(32)

Assuming that \( \overline{v_{in}^2} \) is independent of frequency (white noise)

\[ V_{onrms}^2 = f_i^2 f_i^2 \overline{v_{in}^2} \int_0^\infty \frac{(f_i^2 + f^2)}{f_i^2 + f^2} \overline{v_{in}^2} \, df \]  

(33)

The function to be integrated can be expressed as the sum of two terms with the form

\[ \frac{A_i f + B_i}{f_i^2 + f^2} \]
\[
\frac{\left(f_i^2 + f^2\right)}{\left(f_i^2 + f^2\right)} = \frac{A_1 f + B_1}{(f_i^2 + f^2)} + \frac{A_2 f + B_2}{(f_i^2 + f^2)} = \frac{(A_1 f + B_1)(f_i^2 + f^2) + (A_2 f + B_2)(f_i^2 + f^2)}{(f_i^2 + f^2)(f_i^2 + f^2)}
\]

Equating the coefficients of the terms of the same order:

\[
\begin{align*}
A_1 &= A_2 = 0 \\
B_1 &= \frac{f_i^2 - f_i^2}{f_i^2 - f_i^2} \\
B_2 &= -\frac{f_i^2 - f_i^2}{f_i^2 - f_i^2}
\end{align*}
\quad (34)
\]

Replacing (34) in (33),

\[
V_{onrms}^2 = \frac{f_i^2 f_2^2}{f_i^2} V_{in}^2 \int_0^\infty \frac{B_1}{(f_i^2 + f^2)} + \frac{B_2}{(f_i^2 + f^2)} df 
\quad (35)
\]

Applying to (35) the integration rule

\[
\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctg \left(\frac{x}{a}\right) + C
\]

it results:

\[
V_{onrms}^2 = \frac{f_1^2 f_2^2}{f_i^2} \left[ \pi \left( \frac{1}{f_i} \left( \frac{f_i^2 - f_i^2}{f_i^2 - f_i^2} \right) - \frac{1}{f_2} \left( \frac{f_i^2 - f_i^2}{f_i^2 - f_i^2} \right) \right) \right] V_{in}^2 
\quad (36)
\]

\[
V_{onrms}^2 = \frac{f_1 f_2}{f_i^2} \left[ \frac{\pi}{2} f_1 f_2 \left( f_i^2 + f_1 f_2 \right) \right] V_{in}^2
\]

Taking into account that \( f_i = \frac{1}{2\pi\tau_i} \)

\[
V_{onrms}^2 = \frac{1}{\tau_1 + \tau_2} \left( 1 + \frac{\tau_i^2}{\tau_1 \tau_2} \right) V_{in}^2 
\quad (37)
\]
This is the output noise of the transimpedance amplifier, for a white noise source considering the source capacitance and neglecting the OA input capacitance.

Comparing (27) with (26) it results:

\[ \tau_1 + \tau_2 = R_f \left( C_f + \frac{C_d}{A_0} \right) \]  
(38)

and

\[ \tau_1 \tau_2 = R_f C_d \left( \frac{\tau_a}{A_0} \right) \left( \frac{\tau_z}{B} \right) \]  
(39)

where \( B = A_0/\tau_a \) is the gain-bandwidth product and \( \tau_z = R_f C_d \).

Replacing (39) in (37)

\[ V_{\text{rms}}^2 = \frac{1}{\tau_1 + \tau_2} \left( 1 + B \tau_z \right) \frac{v_{in}^2}{4} \]  
(40)

Since \( B \tau_z \gg 1 \)

\[ V_{\text{rms}}^2 = \frac{1}{\tau_1 + \tau_2} B \tau_z \frac{v_{in}^2}{4} \]  
(41)

If we assume that \( v_{in}^2 \) is due only to the thermal noise of the amplifier input transistor,

\[ v_{in}^2 = 4kT \frac{\gamma}{g_m} \left( V^2 \text{ Hz}^{-1} \right) \]  
(42)

where \( k \) is the Boltzmann constant, \( T \) is the absolute temperature and \( g_m \) is the transconductance of the input transistor. At ambient temperature, \( T = 300 \text{ K} \),

\[ 4kT = 4kT = 1.66 \times 10^{-20} \text{ VC} \]

Coefficient \( \gamma \) is 2/3 for long-channel transistors in saturation, and has higher values for short-channel transistors. In an approximate noise analysis it is common practice to consider \( \gamma = 1 \) [1], a procedure that will be followed in this report.

Replacing (42) in (41) and making \( \gamma = 1 \)
\[ V_{on\text{rms}}^2 = kT \frac{1}{B} \frac{g_{m1} \tau_z}{\tau_1 + \tau_2} \]  

(43)

Taking into consideration (38):

\[ V_{on\text{rms}} = \left[ kT \frac{1}{g_{m1}} \left( \frac{1}{1 + \frac{A_0 C_f}{C_d}} \right) \tau_a \right]^{1/2} A_0 \]  

(44)

Thus, the rms noise voltage is directly proportional to the square root of the bandwidth of the amplifier and inversely proportional to the square root the transcondutance of the input transistor of the AO.

As an example we consider the same values in section 6, with \( \tau_1/\tau_2 = 5 \) and \( \tau_2/\tau_d = 0.1 \):

\[ A_0 = 10^3 \quad R_f = 900 \text{ k}\Omega \quad C_f = 12.2 \text{ fF} \quad \tau_a = 9 \text{ ns} \quad \frac{A_0 C_f}{C_d} = 1.22 \quad \tau_a/\tau_d = 0.22 \]

Assuming an NMOS input transistor, the transconductance is

\[ g_{m1} = 2 \sqrt{K_n \frac{W}{L} I_D} \quad K_n = \frac{1}{2} \mu_n C_{ox} \]  

(45)

where \( \mu_n \) is the electron mobility, \( W/L \) is the ratio between the width and the length of the channel (aspect ratio), \( C_{ox} \) is the capacitance per unit area of the capacitor formed by the gate electrode and the channel and \( I_D \) is the bias current.

It can be seen that high values of \( g_{m1} \) require a high \( W/L \) and a high \( I_D \) meaning large areas and high power consumption. Therefore, \( g_{m1} \) is selected to achieve the target noise values and excessive values should be avoided.

In project PET it is considered that the noise voltage should be \( V_{on\text{rms}} \leq 5 \text{ mV} \). From Fig. 20 it can be seen that for \( \tau_a/\tau_d = 0.22 \) the noise level is bellow 5 mV for \( g_{m1} > 8 \text{ mS} \) approximately.

With \( g_{m1} = 10 \text{ mS} \), we obtain \( V_{on\text{rms}} \approx 4.6 \text{ mV} \).

The results obtained were verified by SPICE simulation using the circuit in Fig. 21, where V3 is at the position of the equivalent input noise source and has frequency sweeping.
The model for the thermal noise of a MOS transistor is a current source in parallel with the channel, with spectral density \( \overline{i_n^2} = 4kTg_{m1} \), assuming that \( \gamma_1 = 1 \) [1,7]. This source is modeled in Fig. 21 by a resistor \( R_2 = 1/g_{m1} \), which produces the same noise spectral density; this resistance, together with \( C_3 \) establishes the OA dominant pole (\( \tau_a = R_2C_3 \)).

![NOISE IN ONE STAGE AMPLIFIER: \( \tau_1/\tau_2 = 5 \), \( \tau_2/\tau_d = 0.1 \)](image)

Fig. 20 – Output noise voltage of the transimpedance amplifier.

![SPICE circuit used to simulate the amplifier noise.](image)

Fig. 21 – SPICE circuit used to simulate the amplifier noise.
$G_1$ is a voltage controlled current source which corresponds to the incremental model of the input transistor. The constant GAIN is the value of the transconductance of the source $G_1$ and corresponds to the value of $g_{m1}$. GAIN1 is a voltage gain block that defines the low frequency gain of the amplifier.

The rms noise is calculated by

$$V_{on \ rms} = \sqrt{\int_{0}^{\Delta f} \frac{v_{on}^2}{\Delta f}} df$$  \hspace{1cm} (46)$$

Theoretically the value of $\Delta f$ is infinite. For practical purposes this value is determined by inspection of the spectral distribution of $v_{on}^2$. The value obtained is:

$$V_{on \ rms} = 4.6 \text{ mV}$$

which validates the result obtained above using equation (44).

Low noise demands high values of transconductance which points to a NMOS input transistor with large width. It should be noted, however, that increasing the width of the transistor increases the capacitances, which, in turn, lowers the bandwidth of the amplifier. Thus, there is a limit in the noise reduction by this procedure.
8. TWO-STAGE TRANSIMPEDEANCE AMPLIFIER USING IDEAL OAs

As we have concluded in previous sections, it is not possible to design an amplifier with the characteristics required in project PET, namely a high gain and fast response, with only one stage. This is so because the values of the feedback capacitance $C_f$ involved are of the order of the parasitic capacitances and a high gain-bandwidth product is needed. Therefore, a two-stage amplifier should be considered.

In order the have a first insight, we consider a 2-stage transimpedance amplifier with the circuit in Fig. 22 using ideal OAs. The transfer function is:

$$\frac{V_o}{I_{dm} R_1 R_2 R_3} = \frac{\tau_d}{(1 + s \tau_1)(1 + s \tau_2)(1 + s \tau_d)}$$

(47)

where $\tau_1 = R_1 C_1$ and $\tau_2 = R_2 C_2$.

This transfer function is formally identical to (20) for the case of one stage amplifier with dominant pole. The equations for $v_o(t)$ are presented in Table 3 for the different cases of the parameters $\tau_1$, $\tau_2$ and $\tau_d$, and time-domain plots are represented in Fig. 23.

Simulations have shown that low values of the peaking time can be obtained with capacitances well above the parasitic capacitances when $\tau_1/\tau_2 \approx 1$ and $\tau_2/\tau_d << 1$.

As an example we consider that $V_{om} = 1$ V and assume that $C_2 = 200$ fF, $R_2/R_3 = 20$ and $R_3 = 1$ kΩ. Using the plots in Fig. 23, we obtain:

$R_1 \approx 30.7$ kΩ  $C_1 \approx 130$ fF  $R_2 = 20$ kΩ  $C_2 \approx 200$ fF  $t_m = 16$ ns
Table 3 – Response of 2-stage transimpedance amplifier using ideal OAs.

<table>
<thead>
<tr>
<th>Frequency Domain</th>
<th>Condition</th>
<th>Time Domain</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{V_o}{I_{dm} R_1 R_2^2 R_3^2} = \frac{\tau_d}{(1+s \tau_1)(1+s \tau_2)(1+s \tau_d)} )</td>
<td>( \tau_1 \neq \tau_2 ) ( \tau_1 \neq \tau_d ) ( \tau_2 \neq \tau_d )</td>
<td>(- \frac{V_o}{I_{dm} R_1 R_2^2 R_3^2} = \frac{\tau_1 \tau_d e^{-\frac{t}{\tau_1}}}{(\tau_1-\tau_2)(\tau_1-\tau_d)} - \frac{\tau_2 \tau_d e^{-\frac{t}{\tau_2}}}{(\tau_1-\tau_2)(\tau_2-\tau_d)} + \frac{\tau_d^2 e^{-\frac{t}{\tau_d}}}{(\tau_1-\tau_d)(\tau_2-\tau_d)} )</td>
</tr>
<tr>
<td>( \tau_1 \neq \tau_2 ) ( \tau_1 \neq \tau_d ) ( \tau_2 = \tau_d )</td>
<td>(- \frac{V_o}{I_{dm} R_1 R_2^2 R_3^2} = \frac{1}{(\tau_1-\tau_d)^2} \left[ \tau_1 \tau_d e^{-\frac{t}{\tau_1}} + (\tau_d-\tau_1) t e^{-\frac{t}{\tau_d}} \right] )</td>
<td></td>
</tr>
<tr>
<td>( \tau_1 \neq \tau_2 ) ( \tau_1 = \tau_d ) ( \tau_2 \neq \tau_d )</td>
<td>(- \frac{V_o}{I_{dm} R_1 R_2^2 R_3^2} = \frac{1}{(\tau_2-\tau_d)^2} \left[ \tau_2 \tau_d e^{-\frac{t}{\tau_2}} + (\tau_d-\tau_2) t e^{-\frac{t}{\tau_d}} \right] )</td>
<td></td>
</tr>
<tr>
<td>( \tau_1 = \tau_2 = \tau ) ( \tau_1 \neq \tau_d ) ( \tau_2 \neq \tau_d )</td>
<td>(- \frac{V_o}{I_{dm} R_1 R_2^2 R_3^2} = \frac{1}{(\tau-\tau_d)^2} \left[ \tau_d^2 e^{-\frac{t}{\tau_d}} + \frac{\tau-d}{\tau_d} t e^{-\frac{t}{\tau_d}} \right] )</td>
<td></td>
</tr>
<tr>
<td>( \tau_1 = \tau_2 = \tau_d )</td>
<td>(- \frac{V_o}{I_{dm} R_1 R_2^2 R_3^2} = \frac{1}{2} \left( \frac{\tau_d}{\tau_d} \right)^2 e^{-\frac{t}{\tau_d}} )</td>
<td></td>
</tr>
</tbody>
</table>
These results were verified by SPICE simulation. The circuit used is presented in Fig. 24. The blocks E1 and E2 are voltage controlled voltage sources with a high gain ($10^5$), which model the ideal OAs. The block GAIN1 is a buffer with voltage gain set to -1 to invert the output signal since it is negative.

The results from SPICE simulation, presented in Fig. 25, are:

$$V_{om} = 1 \text{ V}, \ t_m = 16 \text{ ns}$$

which agree with the theoretical results obtained above.
Thus, we can conclude from this first approximation of a 2-stage amplifier, that the capacitances involved are now of the order of hundreds of fF, well above the parasitic capacitances, and that the values of the resistances changed from the order of MΩ to that of tens of kΩ, which is preferable (smaller area).

![2-STAGE TRANSIMPEDEANCE AMPLIFIER WITH IDEAL OAs: SPICE SIMULATION](image)

**Fig. 25** – SPICE simulation of the response of the 2-stage transimpedance amplifier ($\tau_1/\tau_d = 0.1$).
9. CONCLUSIONS

In this report a review was presented of the transimpedance amplifier used in the front-end of radiation detectors (often referred to as charge sensitive amplifier). It was assumed that the amplifier specifications are those of the project PET [1,2].

The analysis presented here clarifies the influence of the amplifier parameters, namely the gain-bandwidth product, the feedback capacitance and resistance, and the APD capacitance at the amplifier input. The performance of the amplifier was characterized by the amplitude and the peaking time of the output voltage pulse.

Considering the project PET specifications we conclude that it is impossible to design the amplifier with only one stage, due to:

\[ i) \] the low values of the feedback capacitance are of the same order of those of the parasitic capacitances (tens of fF).

\[ ii) \] the high values of the gain-bandwidth product required, of the order of tens of GHz, exceed the technology limit of about 10 GHz.

An architecture with 2 stages allows a gain-bandwidth product significantly higher, an increase of the values of the capacitances and a reduction of the values of the resistances. A preliminary examination of a 2-stage amplifier showed that the order of the resistances changed from MΩ to tens of kΩ, and the capacitances increased from tens to hundreds of fF.

A stability analysis was presented which shows that, as a second order system, the amplifier is stable and can be designed with a suitable phase margin.

A noise analysis was presented, considering the input noise source of the operational amplifier. The output rms noise voltage is directly proportional to the square root of the bandwidth of the amplifier and inversely proportional to the square root of the transcondutance of the input transistor (assuming that the noise contribution of this transistor is dominant). Values of the order of 10 mS are needed to obtain an output rms noise voltage below 5 mV, specified in project PET [1,2].

The high transcondutance values required are more easily obtained with an NMOS input transistor. It should be noted that increasing the width of the transistor increases the capacitances, which lowers the amplifier bandwidth. This sets a limit to the increase of transcondutance.

The use of an input NMOS transistor has also the advantage that the input signal is referenced to ground instead of to the supply rail (as it would be in the case of a PMOS transistor), therefore avoiding the effect of fluctuations of the supply voltage.

Future work will concentrate on the study of a 2-stage amplifier. Different architectures will be considered, and the results will be presented in the next progress report.
In the noise analysis presented here two assumptions have been made, which may not be applicable:

- the input capacitance of the OA was neglected;
- the noise of the feedback resistor $R_f$ was neglected.

Future work should investigate the effect of considering the noise generated by $R_f$, since this may be dominant if a very wide input transistor is used in the OA. The effect of the OA input capacitance should also be investigated.
REFERENCES


