HIDDEN MARKOV MODELS BASED SUPERVISION
OF SENSOR FAULTS: APPLICATION TO
NEUROMUSCULAR BLOCKADE CONTROL

H. Magalhães* J.M. Lemos** T. Mendonça*

* Departamento de Matemática Aplicada
Faculdade de Ciências - Universidade do Porto
Rua do Campo Alegre, 687, 4169-007 Porto, Portugal
Telephone: + 351-220100802
Fax: + 351-220100809
E-mail: {hfmagalh, tmendo} @fc.up.pt

** INESC-ID/IST
Rua Alves Redol, 9, 1000-029 Lisboa, Portugal
Fax: +351-213145843
E-mail: jlml@inesc.pt

Abstract: The problem of embedding sensor fault tolerance in the control of neuromuscular blockade is considered. A hidden Markov model whose state, assumed unavailable for direct observation, is estimated using a nonlinear filtering algorithm, is proposed for detecting the faults.

Keywords: Hidden Markov Models, Nonlinear Filtering, Sensor Faults, Neuromuscular Blockade Control, Fault detection and Isolation

1. INTRODUCTION

Physiological variables are associated to systems whose dynamic behavior is highly uncertain and usually time varying. Furthermore, their measurement may be unreliable and susceptible to random interruptions, due to the complexity and indirect principles upon which they rely. If these measurements are used for closing a control loop, their interruptions may induce strong transients or even instability. When interruptions (sensor faults) occur, two practical problems must be solved:

- Detection of the sensor fault occurrence;
- Signal reconstruction during the fault.

Muscle relaxant drugs are frequently given during surgical operations. The non-depolarising types of muscle relaxant act by blocking the neuromuscular transmission, thereby producing muscle paralysis. The level of muscle relaxation is measured from an evoked EMG at the hand induced by external electrical stimulation. This measurement procedure is prone to failure due to the interaction between technological devices and physiological systems. Indeed, measurement faults have been reported in practice. Figure 1 shows one example of the occurrence of measurement faults in a neuromuscular blockade record, obtained in a clinical environment with an open-loop system.

In (Lemos et al., 2003) the problem is solved using a bayesian algorithm by assuming that the sensor faults occur independently of each other. An alternative is to take advantage of interdependencies between the successive states of “fault” or “no fault”.

Hidden Markov models (HMM) provide an adequate framework for this approach. HMM’s are Markov chains whose states are not directly accessible for measure but generate observable data (Rabiner, 1989; Jelinek, 1997). As such they sig-
significantly generalize the class of stochastic processes which can be modelled, avoiding much complication on the basic structure defined by Markov chains. An HMM model is defined by

- A state space (discrete);
- A probability distribution of transition between states, according to a Markov model;
- An output alphabet build up with the possible observable “symbols”
- An output probability distribution which relates the output symbols with the Markov chain states.

In relation to this structure, two basic classes of problems may be considered:

- **Identification:** From sets of consecutive observations, estimate the probability distributions which define the HMM.
- **Estimation:** From sets of consecutive observations, estimate, in a recursive way, the state in which the HMM is at each time. This can be subdivided in prediction, filtering and smoothing with the filtering problem being the one of interest in the context of this paper.

These problems are solved resorting to Bayesian methods, with the Viterbi, Baum and Baum-Welch algorithms playing a central role (Rabiner, 1989). The classical applications of HMM were in digital communications (Viterbi and Omura, 1979) and speech recognition (Jelinek, 1997; Rabiner, 1989). In speech recognition, having observed the acoustic processor string \( A \), the aim is to find the word string that is the most likely to have produced \( A \). This can be rephrased in a fault detection and isolation (FDI) context and, actually, there is a growing interest on applying methods to FDI and monitoring. Examples range from robotics (Aycard and Washington, 2000) to medical diagnosis (Al-Ani et al., 2000), target tracking in image processing (Marques and Lemos, 2001; Marques et al., 2002), complex communication systems (Smyth, 1994a) and machine maintenance (Bunks et al., 2000). In (Robertson et al., 2003) the modelling of a complex phenomena with a HMM with just 4 states is performed, paralleling in this respect the situation considered in this paper. In (Smyth, 1994a) it is shown that a pattern recognition system combined with a finite state HMM provides a useful method for modelling temporal context in continuous monitoring. The parameters of the Markov model are derived from “macroscopic” failure statistics such as the mean time between failures. In (Lemos et al., 1999) a supervisor algorithm for application in switching reconfigurable control of plants with sparse in time variations is proposed. The possible outcomes of the parameters configuring the plant dynamics are described by a HMM whose state is estimated from plant i/o data.

In this paper an HMM based supervisor of sensor faults is developed and applied to automatic control of neuromuscular blockade. The supervision algorithm assumes that the sensor is either in the state of normal operation, in which case its output is the correct signal plus Gaussian noise, or faulty, in which case the output is a constant probability distribution over the range of possible values. According to the model considered, the transition between the states “normal” and “faulty” is governed by a HMM. The state is inferred from the observations by using a Baum type algorithm and the transition probabilities are estimated by the Viterbi training algorithm. When a fault is detected, the neuromuscular blockade signal is reconstructed using a suitable model (Lemos et al., 2003). The contributions of this paper consist in the integrated supervisor/controller algorithm and its simulation testing in the neuromuscular blockade problem.

## 2. NEUROMUSCULAR BLOCKADE CONTROL

During surgical procedure patients are usually under general anaesthesia, defined as the lack of response and recall to noxious stimuli, reflected in loss of consciousness, pain insensitivity and muscle paralysis. The control of the neuromuscular blockade provides a good illustration of the main features and inherent constraints associated with the control of physiological variables for optimal therapy and is used as a case study. Such control system is characterized by a large uncertainty of the dynamic behaviour as well as the need of a very high degree of reliability and robustness with regard to inter and intra-individual variability of the patients responses (Lago et al., 1998).
The dynamic response of *atracurium* may be modelled by a Wiener structure (Lago et al., 1998; Weatherley et al., 1983) consisting of a linear part in series with a nonlinear one (Figure 2). The linear part is a simple compartmental model which relates the infused dose of *atracurium* with the plasma concentration $c_p(t)$ and with the effect compartment concentration $c_e(t)$, which is related to the induced pharmacodynamic response $r(t)$ by a nonlinear memoryless output function (Hill equation):

$$r(t) = \frac{100C_{50}^\beta}{C_{50}^\beta + c_e^\beta(t)}$$ (1)

where $C_{50}$ and $\beta$ are patient dependent parameters and $r(t)$ is given in %. High values of the infusion rate $u(t)$ (manipulated variable) tend to increase $c_p(t)$, since these variables are related by a linear model. From (1) it is seen that an high level of $u(t)$ will decrease $r(t)$. Instead, when $u(t) = 0$, $c_p(t)$ will tend to zero and $r(t)$ will approach 100%. The final model for *atracurium* has 8 parameters, which are assumed to follow a multidimensional log-normal probability distribution (Lago et al., 1998). To achieve a high level of neuromuscular blockade in a short time, a bolus of *atracurium* is always administered in the beginning of a surgery. After the administration of the bolus, the level of the muscular blockade decreases very quickly, and full muscle paralysis is induced in a few minutes. Following that initial period, the control objective is to follow a reference profile with a final constant value $ref \equiv ref_0$. For that purpose, a family of $N$ non-linear dynamic models, $M_j$, has been generated using the probabilistic model for *atracurium* discussed previously. Each model $M_j$ has associated a modified PID controller $C_j$ (Mendonça and Lago, 1998) which solves the desired tracking problem.

3. SENSOR FAULTS

3.1 Model

The model generating the faults can be considered as in Figure 3. At each time $t$ the neuromuscular blockade level is obtained from the state of a finite state Markov chain with states $S_i$, $i = 1, 2$. Let $S_1$ be the state corresponding to the occurrence of a normal measurement and let $S_2$ be the state corresponding to the occurrence of a sensor fault, i.e. an “outlier”. Let $P(t) \equiv [P_1(t), P_2(t)]$ be the vector of a priori probabilities of the states of the Markov chain at time $t$, i.e. $P_i(t)$ represents the probability that the Markov chain is in state $S_i$ at time $t$. Let $A = [a_{ij}], i, j = 1, 2$ be the state transition matrix such that

$$P(t) = A’P(t - 1).$$

Two different examples are given in this paper. The first one assumes that $A$ is known while in the second the Viterbi training algorithm (Rabiner, 1989), based on the computation of the occurrence frequency from state $S_i$ to state $S_j$, is used to estimate on-line the state transition matrix $A$.

3.2 Nonlinear filtering

Let $r(t)$ denote the observation made at discrete time $t$ and $x(t)$ be the corresponding true value of the variable to measure. It is assumed that $x_{min} \leq x(t) \leq x_{max}$. Under state $S_1$, the observation $r(t)$ is equal to the value of the variable to measure, $x(t)$, added by zero mean white Gaussian noise of (constant) variance $\sigma^2$, denoted $e(t)$

$$r(t) = x(t) + e(t).$$ (2)

Under state $S_2$, a measure interruption occurs. In this case, the observation is no longer related to the variable being measured but, instead, is given by a random variable $\eta(t)$ with a probability density function (p.d.f.) which presents “heavy tails”, thereby explaining the frequent occurrence of high values of $\eta$. Once the signal $x(t)$ to be measured is confined between the values $x_{min}$ and $x_{max}$, it is reasonable to assume that the p.d.f. of $\eta$ is uniform between these two values. This is the case of neuromuscular blockade where $r(t)$ takes values between 0% (full paralysis) and 100% (normal muscular activity). According to a nonlinear filtering approach (Lemos et al., 1999),
in order to detect that a given observation is actually noise, the probability of both states given the observations is computed. Let $P_i(t-1), i = 1, 2$ be the probability that the Markov chain is in state $S_i$ at time $t-1$, given the observations up to time $t-1$, and $P_i(t|t-1)$ the probability of state $S_i$ at time $t$, given observations up to time $t-1$. Using Bayes law, these are recursively propagated in time by the operations of:

A) Prediction

$$P_j(t|t-1) = \sum_{i=1}^{2} a_{ij} P_i(t-1), \quad j = 1, 2 \quad (3)$$

B) Filtering

Given the model of observations when state $S_1$ holds,

$$P_1(t) = C \frac{1}{\sigma_t \sqrt{2\pi}} e^{-\frac{(t-\mu)^2}{2\sigma_t^2}} P_1(t|t-1) \quad (4)$$

where $C$ is a normalizing constant. When $S_2$ holds,

$$P_2(t) = \frac{1}{x_{max} - x_{min}} P_2(t|t-1) \quad (5)$$

with $C$ the same constant as in (4).

For computing (4), the value of $x(t)$ is needed. Since this is unknown, it is replaced by a convenient estimate. To this end, several possibilities may be envisaged. If switched multiple model control is implemented as in (Mendonça et al., 2002), an estimate of the dynamics generating data is available. This can be used to compute an approximation of $x(t)$. Another possibility is to estimate a predictor on the basis of an ARMAX model by using the reconstructed signal. Both probabilities $P_1(t)$ and $P_2(t)$ are then compared. If

$$\frac{P_2(t)}{P_1(t)} > 1$$

it is decided that a sensor fault has occurred. In this case, the observation $r(t)$ is discarded and replaced by a forecast $\hat{x}(t)$ of the true value $x(t)$, made from previous observations.

4. SIMULATION EXAMPLES

In this section, simulation examples are presented in which the above nonlinear filtering algorithm is used to estimate the hidden Markov chain state and to detect the sensor faults.

4.1 Example 1

Figures 4-6 show simulation results obtained with neuromuscular blockade control in which the state transition matrix $A$ is known and equal to

$$A = \begin{bmatrix} 0.9 & 0.1 \\ 0.8 & 0.2 \end{bmatrix}.$$ 

As can be seen in Figure 6, the number of state estimation errors is very low. Therefore the major “outliers” have been removed and the reference is perfectly tracked (see Figure 5).

Fig. 4. Example 1. Sensor output measurement $r(t)$ [%].

Fig. 5. Example 1. Filtered $r(t)$ [%] and control action $u(t)$ [µg/kg/min].

4.2 Example 2

Figures 7-9 illustrate simulation results obtained with neuromuscular blockade control in which the state transition matrix $A$ is assumed to be unknown and is estimated by the Viterbi training algorithm. Let the real matrix $A$ be

$$A = \begin{bmatrix} 0.95 & 0.05 \\ 0.8 & 0.2 \end{bmatrix}.$$ 

In order to reduce the estimates variations and to avoid zero probability estimates for transitions
Fig. 6. Example 1. Estimated state and true state of the hidden Markov chain.

Fig. 7. Example 2: Sensor output measurement \( r(t) \) [%].

Fig. 8. Example 2. Filtered \( r(t) \) [%] and control action \( u(t) \) [\( \mu g/kg/min \)] with very low probability, a low pass-filter with transfer function

\[
H(z^{-1}) = \frac{0.005}{1 - 0.995z^{-1}}
\]

has been implemented.

As can be seen in Figure 9, the number of state estimation errors is once again very low, although neither all the elements of the estimated matrix \( \hat{A} \) converge to their true values,

\[
\hat{A}(t = 120\text{min}) = \begin{bmatrix} 0.93 & 0.07 \\ 0.93 & 0.07 \end{bmatrix}.
\]

In order to test the non-linear filtering algorithm, a Monte Carlo simulation study was made. The simulation conditions were the same as in Example 2 and 100 trials with different noise and outliers sequences were considered. The mean number of errors obtained was 1.96 with a standard deviation of 0.94. Furthermore, to know how the estimated transition probabilities can affect the number of state estimation errors, a sensitivity study to the entries of the estimated matrix \( \hat{A} \) was made. Let the real matrix \( A \) be the same as in Example 2 and let the estimated matrix \( \hat{A} \) be a function of two parameters \( \epsilon_1 \) and \( \epsilon_2 \),

\[
\hat{A}(\epsilon_1, \epsilon_2) = \begin{bmatrix} 0.95 - \epsilon_1 & 0.05 + \epsilon_1 \\ 0.8 - \epsilon_2 & 0.2 + \epsilon_2 \end{bmatrix}.
\]

The results obtained in Figure 10 show that the number of state estimation errors is very low for a wide range of values of \( \epsilon_1 \) and \( \epsilon_2 \), proving the robustness of the non-linear filtering algorithm.
5. CONCLUSIONS

The problem of embedding sensor fault tolerance in the control of neuromuscular blockade has been considered from a unified point of view. A hidden Markov model whose state, assumed unavailable for direct observation, is estimated using a nonlinear filtering algorithm, has been proposed for modelling fault occurrence. Two simulation examples were presented and the robustness of the nonlinear filtering algorithm has been firmly established. The HMM approach is a step forward to the bayesian algorithm that has been tested in a clinical environment. The experience and the good clinical results obtained so far with this algorithm suggests that a good performance of the HMM technique in real data will be attained.

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6. REFERENCES


