Impedance Evaluation of Integrated Circular Spiral Inductors

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Abstract
The impedance of integrated circular spiral inductors is evaluated by using the moment method for the quasi-static axisymmetric magnetic field approximation. Comparison with experimental and numerical results shows that this is an efficient and accurate approach.

1 Introduction
This paper is a contribution to the numerical evaluation of the impedance, inductance and resistance, for planar circular spiral on-chip inductors. For typical geometric dimensions of integrated spiral inductors, at very high frequency values (typically greater than 10 GHz), full wave electromagnetic numerical approaches must be applied [1]. Otherwise, for a quasi-static point of view, capacitance, inductance and resistance effects may be considered as lumped elements [2]. Empirical or semi-empirical approaches have been adopted [2], [4]. These representations, however, can not predict results for different configurations.

In this paper, field equations are solved in the frequency domain where capacitance effects may be neglected but eddy currents inside conductors are taken into account. The moment method [5] is developed using a linear operator obtained from the volume integral formulation for the magnetic vector potential [3]. The approach is based on a 2D axisymmetric field configuration assuming the concentric-ring approximation of circular spiral inductors [1]. Results are compared with experimental and numerical results presented in the literature [1] or given by other electromagnetic field solvers [6].

2 Magnetic Field Formulation
The magnetic field problem is solved in the frequency domain by using the phasors of the field quantities. The circular spiral inductor is approximated to the concentric-ring structure with N rings constituted by a conducting and non-magnetic medium represented in Fig.1. This assumption allows the 2D axisymmetric configuration for the field, assuming that the current density and the magnetic vector potential are vectors with only one component along the azimuthal direction, J, A, respectively. Instead of A it is convenient to use the flux function, ψ, defined by:

$$\psi = 2 \pi \rho A \quad \text{and} \quad 2 \pi \rho J = -j \omega \sigma \psi + \eta \quad (1)$$

where ρ is the radial co-ordinate, σ is the conductor conductivity, ω is the angular frequency and η = $-2 \pi \rho \sigma \nabla \phi$, φ being the electric scalar potential. For the quasi-static field approximation, η is constant in each ring.

The field equation, $L(\psi) = 0$, may be derived from the volume integral formulation for the flux function, the linear operator, $L$, being defined as:

$$L(\psi) = \psi + j \rho^2 \int_S \psi G dS - \int_S (\mu_0 \eta) G dS, \quad \rho^2 = \omega \mu_0 \sigma \quad (2)$$

where S is the cross section of the concentric-ring inductor wire and G is the appropriate axisymmetric Green function representing the flux function originated by a current in an infinitesimal cross section conducting ring.

3 Moment Method
The moment method [5] may be applied to solve the field equation. For this purpose, the cross section of the inductor, $S$, is partitioned into rectangular elements in the ρ direction considering that the thickness t is small enough. The following matrix equation is obtained:

$$\begin{bmatrix} C & H \end{bmatrix} \begin{bmatrix} \psi \\ \mu_0 \eta \end{bmatrix} = \begin{bmatrix} 0 \\ \mu_0 I \end{bmatrix} \quad (3)$$

The elements of the sub-matrices appearing in (3) are defined by:

$$c_{ij} = \delta_{ij} + j \rho^2 G_{ij} \quad (4)$$

where $\delta_{ij}$ is the Kronecker symbol. If $i \neq j$, then $G_{ij}$ is given by:

$$G_{ij} = \frac{S_f}{2 \pi} \int_0^\pi \frac{\rho_j \cos \phi}{\sqrt{(\rho_j - \rho_i)^2 + 2 \rho_j \rho_i (1 - \cos \phi)}} d\phi \quad (4')$$

$S_f$ being the cross section area of the $f^{th}$ element, $\rho_i$.
and $\rho_i$ the radial co-ordinates of the $i^{th}$ and $j^{th}$ elements, respectively. Otherwise, if $i=j$, an analytical treatment is given for the case where Green function has singularities for coincident observation and treatment is given for the case where Green function is given by arctg($\theta/\nu$). Simpson’s method is a good choice to evaluate numerically the integrals in (4). The remaining sub-matrices in (3) are given by:

$$G_{ij} = \frac{4}{\pi} \int_0^\theta \rho_i \cos \phi \, d\phi$$

with

$$\theta_0 \left( \sqrt{\frac{w_i}{2\cos \theta}} \right) + 2\rho_i^2 (1-\cos \phi) - \sqrt{2\rho_i^2 (1-\cos \phi)} \right) d\theta$$

where $w_i$ is the width of the $i^{th}$ element and $\theta_0$ is given by arctg($\theta/\nu$). Simpson’s method is a good choice to evaluate numerically the integrals in (4). The remaining sub-matrices in (3) are given by:

$$h_{ik} = -\sum_n G_{in}$$

where subscript $n$ refers to the $n^{th}$ element in the $k^{th}$ ring. If the $j^{th}$ element belongs to the $m^{th}$ ring:

$$q_{mj} = -J^2 \pi^2 \ln \left( \frac{\rho_{bj}}{\rho_{aj}} \right)$$

otherwise $q_{mj} = 0$, where $\rho_{bj}$ and $\rho_{aj}$ are the maximum and minimum radial co-ordinates of the $j^{th}$ element, respectively. Finally, if $m=k$:

$$x_{mk} = \ln \left( \frac{\rho_{bm}}{\rho_{am}} \right)$$

otherwise $x_{mk} = 0$, where $\rho_{bm}$ and $\rho_{am}$ are the maximum and minimum radial co-ordinates of the $m^{th}$ ring.

## 4 Numerical Results

Numerical results were obtained for the inductance, $L$, and resistance, $R$. They were derived from the time averaged values of the stored magnetic energy and the dissipation power loss inside the conducting spiral inductor, respectively.

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Table 1 – Results at 2 GHz for three different inductors (Fig. 1). Case A: $N=4$, $r=120 \, \mu m$, $w=13.7 \, \mu m$, $s=10.27 \, \mu m$. Case B: $N=4$, $r=140 \, \mu m$, $w=11.38 \, \mu m$, $s=3 \, \mu m$. Case C: $N=6$, $r=160 \, \mu m$, $w=11.25 \, \mu m$, $s=3 \, \mu m$. The first two columns give the simulated and measured results calculated at the maximum $Q$ with a substrate of $5\Omega$-cm Si described in [1]. In the column “This Work” the results are for 16 elements per ring. The last column is obtained using the ASITIC program [6] for AMS 0.35 $\mu m$ technology, considering the fourth metal layer.

Table 1 presents the results for three different spiral inductors; comparisons with other methods and experimental results are indicated. Fig. 2, (a) and (b), presents the results for one of the configurations showing the convergence behaviour of the results with the number of elements per ring.

## 5 Acknowledgements

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## 6 Literature