Abstract: This paper is concerned with state estimation in physical plants involving transport phenomena with no diffusion. These are described by an hyperbolic partial differential equation where the manipulated variable multiplies the state (bilinear structure). Distributed collector solar fields, where the state is the temperature distribution along the field, provide an example which is treated in detail. Observers of two types are considered. The first relies on the method of characteristics and the second on the orthogonal collocation method. A distinctive feature of the paper consists in the fact that both methods explore a change of time scale, which provides an exact linearization of the plant model. For discrete time approximations, this amounts to the use of a variable sampling interval. The stability of the observer filters is established. The observer algorithms are tested through simulations performed in a detailed physical model of the solar field.

Keywords: State estimation, observers, bilinear systems, distributed collector solar fields, process control, orthogonal collocation, distributed systems.

INTRODUCTION

This paper is concerned with state estimation in physical plants involving transport phenomena with no diffusion. These are described by hyperbolic partial differential equations where the manipulated variable multiplies the state (bilinear structure). Distributed collector solar fields such as the ACUREX field of Plataforma Solar de Almeria (Spain) provide an example which is treated in detail. In this plant (fig. 1), the state is the oil temperature distribution along a pipe located at the focus of the mirror collectors. See (Camacho et al., 1997; Silva et al., 2003) for further details and more complete references.

Observers of two types are considered. The first relies on the method of characteristics (Nakamura, 1991; Dochain et al., 1992; Lefèvre et al., 2000) and the second on the orthogonal collocation method (Rice and Do, 1995). A distinctive feature of the paper consists in the fact that both methods explore a change of time scale, which provides an exact linearization of the plant model. This idea, which derives from the natural dynamics of the plant, was explored in (Silva, 2003; Silva et al., 2003a) for designing adap-
tive control algorithms able to track large reference steps with little overshoot. The observers developed here are thus particularly adequate to match the control algorithms of these class which require a state estimate (Silva, 2003a). For discrete time approximations, this amounts to the use of a variable sampling interval. In this paper, the stability of the observer filters is established. The observer algorithms are tested through simulations performed in a detailed physical model of the solar field.

1. OBSERVER BASED ON THE METHOD OF CHARACTERISTICS

The Method of Characteristics (Nakamura, 1991) is used in this section to yield a discrete estimator of the temperature along the pipe.

1.1 The method of characteristics

Consider the reduced model of the field (Barão et al., 2002):

\[
\frac{\partial T(z,t)}{\partial t} + u(t) \frac{\partial T(z,t)}{\partial z} = \alpha \, R(t)
\]

(1)

where \(T(z,t)\) is the temperature at position \(z \in [0, L]\) measure along the pipe and at time \(t \in [0, +\infty[\), \(u\) is the oil velocity (proportional to flow), taken as the manipulated variable, and \(R\) is a known function of solar radiation. The parameter \(L\) has the value \(L = 180\ m\) and denotes the length of the pipe. The parameter \(\alpha\) depends on the mirror efficiency and on oil specific heat. An estimate of \(\alpha\) valid at the operating point is assumed. The boundary conditions for (1) are as follows: The initial temperature distribution along the pipe \(T(z,0)\) \(0 \leq z \leq L\) is specified, as well as \(T(0,t), t < 0\) which corresponds to the temperature of the oil entering the pipe from the storage tank. According to the method of characteristics, (1) is written in the equivalent form

\[
\begin{cases}
\dot{z} = \alpha R(t) \\
\dot{T} = u(t) 
\end{cases}
\]

(2)

with \(T(0) = T(z,0)\) and \(z(0) = 0\). The second of these equations define the characteristic lines. The first corresponds to the temperature along these lines. Consider a space-time mesh with nodes at points \((z_n, t_k)\), by

\[
\begin{align*}
z_0 &= 0, \quad t_0 = 0 \\
z_n &= z_{n-1} + \Delta z, \quad n = 1, 1, \ldots, N \\
t_k &= t_{k-1} + \Delta t, \quad k = 1, 1, 2, \ldots
\end{align*}
\]

Applying finite differences, and using the notation \(T_n(k) \triangleq T(z_n, t_k)\), yields

\[
T_n(k + 1) = T_n(k) + \alpha R(k) \, \Delta(k)
\]

(3)

Fig. 2. The non-uniform mesh used in the method of characteristics.

The increment in time, \(\Delta(k)\) is made to vary in time according to the equation defining the characteristic lines in (2):

\[
\Delta(k) = \frac{\Delta z}{u(k)}
\]

(4)

Considering a space increment \(\Delta z\) constant in time and given by \(L/N\), yields

\[
\Delta(k) = \frac{L}{N} \frac{1}{u(k)} \quad t_k = t_{k-1} + \Delta(k - 1)
\]

(5)

As shown in fig. 2, the characteristic line for \(T_n(n)\) divides the mesh in two domains: Sub-domain I in which the solutions depend on the initial conditions (temperature along the pipe) and sub-domain II in which the solution depends on the boundary condition (inlet oil temperature). Taking this into consideration, it is possible to derive the following recursive formula

\[
T_n(k) = T_{n-k}(0) + U_0(k-n) + \alpha \sum_{j=1}^{n} R(k-j) \Delta(k-j)
\]

(6)

with

\[
T_n(0) = 0, \quad \eta \neq 1, \ldots, N \\
T_0(\xi) = 0, \quad R(\xi) = 0, \quad \xi \neq 1, 2, \ldots
\]

The crucial point is the use of a time varying sampling time interval, chosen according to the characteristic lines by eq.(5). If a new manipulated variable is taken proportional \(R(k)\Delta(k)\) (or to \(\frac{R(k)}{u(k)}\)), the reduced model of the system becomes exactly linear, and a linear state space observer can then be build. Note that, due to physical constraints, the oil velocity \(u\) is always strictly positive.

1.2 State space Model

From the recursive equation (3) the following linear state space model is build, where \(v = \frac{R(k)}{u(k)}\) denotes the new manipulated variable:

\[
x(k+1) = A \, x(k) + \gamma B \, v(k) + D \, d(k)
\]

(7)
\[ y(k) = C x(k) \]  

where

\begin{align*}
A &= \begin{bmatrix}
0 & \ldots & 0 \\
I_{n-1} & \vdots & \vdots \\
0 & \ldots & 0
\end{bmatrix}, \\
B &= \begin{bmatrix}
1 \\
\vdots \\
1
\end{bmatrix}, \\
C &= \begin{bmatrix}
0 \\
\vdots \\
0
\end{bmatrix}, \\
D &= \begin{bmatrix}
1 \\
\vdots \\
0
\end{bmatrix},
\end{align*}

and in which \( d(k) = T_{in}(k) = T_0(k), x(k) = [T_1 \ T_2 \ \ldots \ \ T_N]^T \), \( x(0) = [T_1(0) \ \ldots \ \ T_N(0)]^T \) and \( \gamma = \alpha L/N \). The observability matrix is given by:

\[ O = \begin{bmatrix}
0 & \ldots & 0 & 1 \\
0 & \ldots & 1 & 0 \\
\vdots & \ddots & \vdots & \vdots \\
1 & \ldots & 0 & 0
\end{bmatrix} \]

and hence the model is completely observable.

### 1.3 Estimator

The temperature along the pipe can be estimated by an asymptotic observer for (7,8), followed by an inversion of the time scale defined by (5). The observer is designed by selecting the observer gain \( K_e \) such that the error dynamics \( A - K_e C \) is a stability matrix.

The temperature in the original time scale is recovered by inverting the time scale change (5).

### 2. OBSERVER BASED IN THE ORTHOGONAL COLLOCATION METHOD

According to the Orthogonal Collocation method (OCM), the nonlinear PDE (1) is reduced in a first step to a set of ordinary differential equations (ODE) (Rice, 1995; Dochain, 1992; Lefèvre, 2000). In the second step, a change of time variable which exactly linearizes (1) is performed. The state of the resulting linear state space model is then estimated with a Luenberger observer.

Both the finite difference method and the OCM yield the approximation of a PDE by a set of ODE’s. The OCM has the advantage of requiring a significantly smaller number of ODE’s to achieve the same level of error (Rice, 1995; Dochain, 1992). However, it implies a difficult choice of the values of a set of parameters such as the number of collocation points or the parameters in orthogonal polynomials. These choices, besides being critical for achieving good results, may require a significant simulation effort before a good adequation to the problem at hand is obtained (Lefèvre, 2000).

### 2.1 The OCM method

The approximate solution of (1) by OCM has the form:

\[ T(z,t) = \sum_{i=0}^{N+1} \varphi_i(z) T_i(t) \]

where the functions \( \varphi_i(z) \) are usually chosen as Lagrange interpolation polynomials verifying:

\[ \varphi_i(z_j) = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases} \]

Here, the \( z_i \) for \( i=1,\ldots, N \) are the interior collocation points and \( z_0 \) and \( z_{N+1} \) are boundary collocation points. The derivative with respect to time of the approximate solution is given by

\[ \frac{\partial T(z,t)}{\partial z} = \sum_{i=0}^{N+1} \frac{d\varphi_i(z)}{dz} T_i(t) \]

Inserting in (1) yields

\[ \sum_{i=0}^{N+1} \varphi_i(z) \frac{dT_i(t)}{dt} = -u \sum_{i=0}^{N+1} \frac{d\varphi_i(z)}{dz} T_i(t) + \alpha R(t) \]

In this way, the PDE (1) is reduced to \( n=N+1 \) ODEs in which the state variables are the temperature at the collocation points. In matricial form, this reads

\[ \dot{\hat{T}} = -u (AT + BT_0) + C \alpha R(t) \]

where \( T = [T_1 \ T_2 \ \ldots \ T_{N+1}]^T \) and

\[ A = \begin{bmatrix}
\varphi'_1(z_1) & \varphi'_2(z_1) & \cdots & \varphi'_{N+1}(z_1) \\
\varphi'_1(z_2) & \varphi'_2(z_2) & \cdots & \varphi'_{N+1}(z_2) \\
\vdots & \vdots & \ddots & \vdots \\
\varphi'_1(z_{N+1}) & \varphi'_2(z_{N+1}) & \cdots & \varphi'_{N+1}(z_{N+1})
\end{bmatrix} \]

\[ B = \begin{bmatrix}
\varphi_0'(z_1) \\
\varphi_0'(z_2) \\
\vdots \\
\varphi_0'(z_{N+1})
\end{bmatrix}, \quad C = \begin{bmatrix}
1 \\
1 \\
\vdots \\
1
\end{bmatrix} \]

in which

\[ \varphi_j'(z_i) \equiv \frac{d\varphi_j(z)}{dz} \big|_{z=z_i} \]

and \( T_0(t) \) is the boundary condition.

The matrix form is formally identical to the one obtained by applying finite differences. The difference consists in the values of matrices \( A \) and \( B \), which depends on the collocation points through the interpolation polynomials. These matrices can be obtained from standard procedures (Henson and Seborg, 1997).

Since the domain in which the interpolation functions are orthogonal is \([0,1]\), the change of variable \( z = xL \) with \( x \in [0,1] \) must be performed, yielding:

\[ \frac{\partial T(x,t)}{\partial t} = -\frac{u(t)}{L} \frac{\partial T(x,t)}{\partial x} + \alpha R(t) \]

For \( N=0, \) this reads

\[ \dot{T} = -\frac{u(t)}{L} T + \frac{u(t)}{L} T_0(t) + \alpha R(t) \]

and there are no interior collocation points, the temperature being computed at \( x = 1 \). This model coin-
cides with the one obtained by finite differences with just one element. For \( N = 1 \) one gets
\[
\dot{T} = -\frac{u(t)}{L} \left[ \begin{array}{ccc}
0 & 1 & 1 \\
-4 & 3 & 0
\end{array} \right] T + \frac{u(t)}{L} \left[ \begin{array}{c}
-1 \\
1
\end{array} \right] T_0 + \left[ \begin{array}{c}
1 \\
1
\end{array} \right] \alpha R
\]
There is now a collocation point at \( \xi = 0.5 \). In this case, the model is no longer coincident with finite differences. For higher values of \( N \) advantage is taken of the interpolation, through the choice of collocation points, thus yielding better approximations.

2.2 Implicit discrete recursive filter

For the model obtained from orthogonal collocation in non-normalized space
\[
\dot{T} = -\frac{u(t)}{L} AT - \frac{u(t)}{L} BT_0(t) + \alpha R(t)
\]
perform the change of time variable
\[
\tau(t) = \int_0^t u(\sigma)d\sigma \quad \frac{d\tau}{dt} = u(t)
\]

Since
\[
\frac{dT}{dt} = \frac{dT}{d\tau} \frac{d\tau}{dt} = \frac{dT}{d\tau} u(t)
\]
the model in transformed time \( \tau \) becomes linear:
\[
\begin{aligned}
\begin{cases}
\frac{d\tau}{d\tau} = -A T(\tau) - B L T_0(t(\tau)) + \alpha C \frac{R(t(\tau))}{u(t(\tau))} \\
T(\tau) = \int_0^\tau \frac{u(\sigma)}{u(\sigma)} d\sigma 
\end{cases}
\end{aligned}
\]
Since the model is now linear, a Luenberger type observer can be used. Denote \( \hat{x} \) the state estimate. It satisfies the equation
\[
\dot{\hat{x}} = -A \hat{x} - B L x_0(t) + \alpha C \frac{R(t(\tau))}{u(t(\tau))} + K_e (y - \hat{y})
\]
\[
\hat{y} \triangleq D \hat{x} = \begin{bmatrix} 0 & 0 & \cdots & 1 \end{bmatrix} \hat{x}
\]
with \( \dot{T} = \hat{x} \quad T_0 = x_0 \quad T(L, y) = y \)

Sampling the filter it is possible to obtain the following recursive estimator:
\[
\hat{x}_k = \left( \frac{I_n}{\Delta \tau} + \frac{A}{L} + K_e D \right)^{-1} \left( \frac{\hat{x}_{k-1}}{\Delta \tau} + K_e x_{nk} - \frac{B}{L} x_0(t) + C \alpha \frac{R_k}{u_k} \right) + \frac{R_k}{u_k} \left( x_{nk} - T_0(t_k) \right)
\]
\[
t_k = t_{k-1} + \frac{\Delta \tau}{u_{k-1}} \quad k = 1, 2, \ldots
\]
with
\[
R_k = R(t_k) \quad x_0(t) \equiv T_0(t_k) \quad x_{nk} \equiv T_n(t_k)
\]

and in which \( n = N + 1 \).

2.3 Error stability

The dynamics of the estimation error is given by
\[
\dot{e}(\tau) = \left( \frac{-A}{L} - k_e D \right) e(\tau)
\]
This is stable iff
\[
A_e = \left( \frac{-A}{L} - k_e D \right) e(\tau)
\]
is a stability matrix. Under this condition
\[
\lim_{\tau \to \infty} e(\tau) = \lim_{t \to \infty} e(t) = 0
\]

since
\[
\tau(t) = \int_0^t u(\sigma)d\sigma
\]
where
\[
0 \leq U_{\min} < u(t) \leq U_{\max} < \infty
\]
and hence
\[
U_{\min} < \frac{\tau}{t} < U_{\max}
\]
which implies that
\[
t \to \infty \quad \text{iff} \quad \tau \to \infty
\]

In conclusion, the origin is the only equilibrium point of the estimation error, and the gain \( k_e \) can be designed such that it is asymptotically stable.

3. RESULTS

The results illustrate the application of the methods to the estimation of the temperature along the pipe in a distributed collector solar field. Simulations are performed with a detailed physical model of the plant.

For the method of characteristics (MC), a linear correction has been applied along the pipe by selecting
\[
K_e = \rho \begin{bmatrix} 1 & 2 & \cdots & N \end{bmatrix}
\]

Here, \( \rho \) is a constant and \( N = \frac{k}{\Delta z} \) is the number of elements considered.

For the method of orthogonal collocation (OCM) 3 collocation points at \( \{0.113, 0.500, 0.887\} \) with \( \alpha = \beta = 0 \) (legendre polynomials). Furthermore, a constant correction at the collocation points is made, with \( K_e = \rho \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \).

All the results have been obtained assuming a constant value of the optical efficiency \( \alpha \). The tests shown hereafter compare the estimates yielded by the observers with actual experimental field measures of the outlet oil temperature and also with simulated values at \( z = 160 m \) and \( z = 90 m \), obtained with the detailed physical model. Figures 3 up to 9 show the results in three different experiments. In each case, the time evolution of the temperature in different points of the pipe is shown. The method as well as the corresponding parameter values are shown in the legends. The actual
temperature at the output is shown also, superimposed on the estimated one. Remark the good agreement between the estimated and measured temperatures.

Figs. 10 through 13 show the temperature inside one point of the pipe each, as estimated by the observer and as "measured" from the nonlinear physical model.

4. CONCLUSIONS

Algorithms for state estimation of transport systems have been presented and illustrated through their application to the estimation of temperature along the pipe of a distributed collector solar field.

The key idea consists in performing a change of time scale, associated to the flow in the plant, which linearises the dominant model. This is combined with methods for approximating the PDE model of the plant, including orthogonal collocation.

These observers naturally suit controllers which resort to the same time scaling.
5. REFERENCES


