Distributed LQ control of water delivery canals

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Abstract

This report addresses the problem in controlling water delivery canals by a decentralized fashion. This problem is widely solved by centralized controllers available nowadays but a decentralized approach can much reduce the complexity of such controllers. An innovator algorithm is presented in this work, derived from Linear Quadratic optimization that is used to compute a coordination between neighboring controllers. These decentralized controllers are, thereby, compared with centralized ones from a performance viewpoint.

keywords: Identification, Linear Quadratic Control, Decentralized Coordination, Canal, Gate, Pool, Offtake, Water Level, Experimental.

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1 Introduction

It is an almost unanimous opinion, water is the gold good of 21st century. Taking this into consideration, the problem of water storage arises and this project is directed towards the resolution of that issue. In a river or any sort of water channels, gates provide some freedom to change water levels along the course. Using these applications and joining a sensor network for water level measurements, gives us an interesting problematic to work on, finding suitable control structures consonant with canal operator desires.

Optimization in water channels is a subject of great interest in the researching environment and has been extensively explored. The freshwater that these canals most often carry presented in lakes and rivers, is a scarce resource and, so, is important to preserve it. There’s about 0.8% of all water in the world that humans need to survive [1], spending it by an efficient way will be necessary on a nearby era. Sooner or later, due to population density increasing this question will be emphasized by the governments of all countries worldwide.

It’s required, mostly, that water waste is minimized and, also, energy spared in the operation of such control systems. They could as well be used to prevent that floods occur more frequently or even that reaches of some river run out of water.

There is some research work and effort in water channels control, ranging from PID controllers to LQ regulators, that use upstream or downstream reach data, but still, the centralized controller yet brings much limitations mainly on maintaining such infrastructures. All algorithms available to us, need a communication platform between the centralized system and local controllers plus power to feed the entire control system.

In today’s time, there are many chaotic climate changes going on that are experienced by us. The difference of precipitation from winter to summer, seems to be growing in such a way that we need to get concerned in storing all water we can, when there’s plenty, and to share it in times of shortage.

Populations are fragile in periods of extreme drought, when serious problems in supplying water are faced. In the Agriculture sector even more difficulties appear since dry summers lead to large economic losses or people in rural areas that miss their average mark of livelihood for the rest of the season. Also, public usage will always comes first. In a near future, water dispensed in Agriculture should decrease, mainly due to the increasing demand in Industry and public usage [1].

In Energy, water is also a critical resource. A dam can have a few Gigawatt of nominal power, which could fed a large population, at least for some periods of the year. This source of energy has great importance because it’s an almost unpaid one. Then, it is essential that floodgates maneuvers may maximize the energy produced when standing before a large number of factors related with water waste.

This project is framed in AQUANET - Decentralized and Reconfigurable Control for Water Delivery Multi-purpose Canal Systems which its efforts fall in refining the actual model for multipurpose hydraulic open-channel systems and developing several decentralized controllers that cooperate in order to achieve a near optimal control in these type of systems. Control structures designed were tested on a pilot plant located at University of Évora. AQUANET project intends to integrate fault tolerant algorithms in the decentralized control network that will be developed for the experiment canal, being that such scope is outside the scope of this work. In parallelism with algorithms used, exists Model Predictive Control techniques which are more flexible and makes embedding of fault tolerant theory easier.

2 Modeling the Canal

This section presents some physical characteristics of the experimental canal used for tests that are important for contextualizing the given problem, and also, some mathematical tools and equations that hydraulic engineers often use to model irrigation canals. Furthermore, a quick procedure to model these systems for controller design through data driven modeling is discussed, which, despite of being simpler provide models that are adequate.

2.1 Physical Aspects of the Canal

The automatic canal used in this study is a component of the facilities owned by NuHCC and located at the University of Évora. In parallel with the automatic canal there is also a traditional canal that is linked to the
previous one, forming a closed circle of water. In this way it is possible to have water always running without disturbing any test. The automatic canal is composed of four linked pools, with a total length of 141 m and an average longitudinal bottom slope of $1.5 \times 10^{-3}$.

Considering the water flow direction, at the end of each pool an undershot gate is placed, that drains to the next pool from below. In total there three undershot gates, being that the last is an overshot gate and lets water flow from above. To feed this canal there is a valve called MONOV AR, at the start, that delivers a maximum constant flow of 90 $l/s$. For simulating irrigations in agriculture and make experiments more realistic, four offtake valves are placed at the end of each pool for draining the respective pool.

Concerning the characteristics of a single pool, all of them have in general a trapezoidal cross-section, as can be seen in figure 1. The bottom is straight with 0.15 m width, sides slope have 1 : 0.15 (V:H) and 0.90 m of height. As seen in figure 2, all gates, $M_1$, $M_2$, $M_3$ and $M_4$, are equipped with position sensors measuring gaps between the bottom of the canal and lower border of each gate that can have an 800 mm maximum amplitude, except for the last, where the measurement is done from the upper bound of the gate with a total of 700 mm fully open, since this one does an overshot drainage. Furthermore, each pool has three water level sensors located upstream, center and downstream (represented by vertical arrows pointing up) for retrieving real time data, doing it with a minimum measurement error of 1 mm. Offtake valves as well as the MONOV AR are equipped with a flowmeter to measure the flow that passes through them. Furthermore, all have a controller for adjusting the opening of the valve for maintaining a desirable flow.

There is one PLC located at the reservoir for dealing with the intake flow and returning readings on it. Other four PLC, one aside to each gate, are responsible for giving commands to his respective offtake and gate and receiving readings on sensors located in the area. All these five PLC are, in turn, connected to another PLC that
centralizes all information and sends it to a SCADA system. Thus with SCADA, it is possible to supervise all levels along the canal, manipulate gates and offtakes and change intake flow. Figure 3 shows the connections involved in this network. The implementation of a MATLAB/SCADA interface within AQUANET project [2], made analysis and controller testing in real time a lot easier, since MATLAB may be used as a fast prototyping tool.

SCADA software also offers some modes of control that can be used by the user. In terms of intake flow, a PI controller is installed at the MONOVAR valve for carrying it up to a desirable value. Another modes of control related with water levels along the canal are available. Each gate has three possible modes: position control, used when direct commands are given; manual control, which turns off all SCADA control; or direct control, that uses four local PID controllers, one per gate with no coordination.

2.2 Linear Model Identification

There are several ways to construct a model for irrigation canals, a subject that has been explored over the years [3], [4] and [5]. A possible approach consists in the use of Saint-Venant equations, a pair of partial differential equations (PDE) that embed mass and momentum conservation. The models of Muskingum [6] or reservoir based models [7] are other possible ways to represent reality on such water delivery systems.

Saint-Venant equations are discussed in this dissertation since the nonlinear model used for tests, relies on them [8]. But it does not imply that linear models used, further ahead, are linearizations from the previous equations. They are obtained through a direct identification data, taken from the nonlinear simulation model. Saint-Venant are a pair of nonlinear partial differential equations and are deduced under the following hypotheses:

- Bed slope of the canal is small enough;
- Water is a homogeneous and an incompressible fluid, so, its density is constant;
- Water flow is unidimensional along longitudinal axis;
- The cross section pressure is hydrostatic;
- There are no lateral discharge;
- The effects of internal viscosity are negligible face to the external friction;

By combining these hypotheses with the conservation of mass and the conservation of momentum principles applied to a stretch of the canal, the following equations are obtained.

\[
\frac{\partial Q(x,t)}{\partial x} + \frac{\partial A(x,t)}{\partial t} = 0
\]

(1)
\[
\frac{\partial Q(x,t)}{\partial t} + \frac{\partial}{\partial x} \left( \frac{Q^2(x,t)}{A(x,t)} \right) + gA(x,t) \frac{\partial h(x,t)}{\partial x} = gA(x,t)(S_0(x) - S_f(x,t))
\] (2)

Here \( Q \) represents the flow of water that passes through a cross-section \( A \) of a pool, subjected to a gravitational acceleration \( g \). Since the canal in question is trapezoidal, \( A \) is composed by his height \( h \), that changes along the space dimension (\( x \)) due to wave propagation, and by his greater and lower widths. \( S_0 \) and \( S_f \) represent the bed slope at the bottom and friction slope from sides, respectively. Details on the canal geometry are described in [4].

Building a linear state-space model, which is largely used hereafter, implies a linearization for the Saint-Venant equations. Conceptually, given a state-space model \( \dot{x}(t) = Ax(t) + Bu(t) \), information taken from the watercourse (Saint-Venant equations) will only model the standalone dynamics, that correspond to matrix \( A \), while matrix \( B \) will emerge mainly in relation to gates dynamics. Furthermore, equations for gates placed along the canal need to be found. There are two types of equations to describe gates behavior. Undershoot gates are modeled by

\[
Q = K_{ds}A \sqrt{2g(h_{before} - h_{after})}
\] (3)

while for overshot gates:

\[
Q = BK_{ds} \sqrt{2g(h_{before} - h_{gate})^3}
\] (4)

in which:

- \( Q \) is the flow drained by each gate;
- \( K_{ds} \) the discharge coefficient;
- \( B \), width of the overshot gate;
- \( A \) reflects the effective area of the orifice for undershot gate;
- \( g \) gravitational acceleration;
- \( h_{before} \) and \( h_{after} \), heights before and after gates;
- \( h_{gate} \), height of the overshot gate.

As mentioned above, combining equations (1), (2), (3) and (4) a state space model in continuous time can be built for the canal. Finally a linearization and a discretization is necessary to find a suitable discrete-time model.

Alternatively, to these methods there is identification. This is an adequate procedure for the purposes of this work, although data from the process is needed to construct a model. Since a model of the canal being studied was made available [8], it is possible to tune a given model with data from it. Other option would be to identify a model, based on the actual experimental canal data.

In this section, it is only discussed identification theory applied to SISO systems but it can be extended to MIMO systems as well, in centralized control. Decentralized control, despite of being globally a problem with MIMO systems, locally they are considered MISO systems, multiple-input to single-output.

For identification there are several approaches that can be taken using parametric methods. First, a parametric model should be selected among several possibilities,

- Autoregressive models with exogenous inputs (ARX);
- Autoregressive moving average models with exogenous inputs (ARMAX);
- Output-error (OE) models;
- FIR models;
- Box-Jenkins (BJ) models;
All of the previous are representations of linear time-invariant systems and particular cases of the equation,

$$ A(q^{-1})y(t) = \frac{B(q^{-1})}{F(q^{-1})}u(t-nk) + \frac{C(q^{-1})}{D(q^{-1})}e(t) $$

(5)

Where,

$$ A(q^{-1}) = 1 + a_1q^{-1} + \ldots + a_{na}q^{-na} $$

$$ B(q^{-1}) = b_1 + b_2q^{-1} + \ldots + b_{nb}q^{-nb+1} $$

$$ C(q^{-1}) = 1 + c_1q^{-1} + \ldots + c_{nc}q^{-nc} $$

$$ D(q^{-1}) = 1 + d_1q^{-1} + \ldots + d_{nd}q^{-nd} $$

$$ F(q^{-1}) = 1 + f_1q^{-1} + \ldots + f_{nf}q^{-nf} $$

and in which $q^{-1}$ is the backward shift operator, $a_i, i = 1, \ldots, na, b_i, i = 1, \ldots, nb, c_i, i = 1, \ldots, nc, d_i, i = 1, \ldots, nd$ and $f_i, i = 1, \ldots, nf$ are real parameters and $na, nb, nc, nd$ and $nf$ are positive integers.

OE models are achieved by making $na, nc$ and $nd$ equal to zero; ARMAX models by making $nf = 0$ and $nd = 0$; FIR models by making all terms equal to zero except $nb$ terms; Box-Jenkins models by canceling only $na$ terms; ARX models by neglecting disturbances influence, making $nc = 0$, $nd = 0$ and $nf = 0$. ARX is simplest, the main differences with respect to all the others is that it models disturbances as a white noise sequence. Another choice are state-space models,

$$ x(k+1) = Ax(k) + Bu(k) $$

$$ y(k) = Cx(k) + Du(k) $$

that are the target models for this work.

Afterwards, a parameter estimation method is required to adjust model parameters to the data collected from the process. In this work, only prediction-error methods are considered since they will be intensively used but others exist, like Maximum Likelihood, Bayes or Markov methods. Prediction-error methods are based on error minimization criterions and, according to [9], represent a way of minimizing the cost function,

$$ V_N(\theta, Z^N) = \frac{1}{N} \sum_{t=1}^{N} \ell(e(t, \theta)) $$

(6)

featuring an optimization problem given by

$$ \hat{\theta}_N(Z^N) = \arg\min_{\theta \in \mathcal{D}_g} V_N(\theta, Z^N) $$

(7)

In (6) and (7), $\theta$ is the vector of parameters to be estimated and $Z^N$, $N$ data points collected from the system to be modeled. Function $\ell(\cdot)$ is any norm in $\mathbb{R}^N$ that can measure prediction errors, $e(t, \theta)$. Special algorithms that use this approach differ in the way they choose different types of norms.

A common and much known method is Least-squares, which adopts a quadratic norm,

$$ \ell(e) = \frac{1}{2}e^2 $$

and is used to identify ARX structures.

In this project, is used ARX for supporting identification and, so, there is no need to extend further in identification theory. After fitting data with a suitable parameterized ARX model, a direct conversion to state-space models is done to obtain a structured model that, later on, can be refined by prediction-error methods. With this approach we can impose the structure of any model previously identified from a process and, then, improve it with PEM. This is a two phase identification that greatly improves final results.
Further ahead, all results from identification performed were obtained through nonlinear simulation model [8] since tests made during this work, acknowledge a good emulation of the physical process. So, this kind of identification will not be discussed in this work.

The main reason for choosing ARX models, having such a variety, was a matter of simplicity and inexistence of stochastic disturbances in the experimental process, implicit on this work. Although in the simulation model [8], noise attached to the data is not white. Numerical issues in the model are responsible for this fact and produce unintentionally such effects. Figure 4(a) shows data obtained from the simulation model and the output of a linear model (ARX) identified from the previous data. It can be seen that the high frequency noise presented in gate 3 and pool 3 data is not covered by the ARX model. In figure 4(b) is presented a correlation analysis for the residue between the simulation model and the linear model within same identification performed in figure 4(a). It can, also, be seen that, in fact, noise is not white since it does not have the form of a white noise correlation function. However, the target process in this work is not the simulation [8] but the experimental canal, located at NuHCC facilities. So, ARX models are still intensively used in this work.

Given this, both ARMAX and Box-Jenkins models do not significantly improve identification, despite of having more degrees of freedom. Although OE models, as mentioned in [9], describe low frequency properties better than ARX, which are more important for control problems. ARX models are specialized in higher frequencies. However, since OE models do not improve much the results, the ARX structure is still preferred.

To perform identification of the process at hand, small variations around an equilibrium value were made on the input, using a pseudo random square wave. The excitation of the input needs to be small in order to avoid nonlinearities. Furthermore, we are dealing with slow processes meaning that a large sampling period (2s) is to be used. A square wave excites a larger range of frequencies at step transitions, being thus convenient the use of such input signals to perform identification. The control period can not be too big because controller would see a much slower process than it actually is which will result in performance loss. Figure 5 illustrates waves propagating along the canal in a normal situation. Time instants, in multiple values of two seconds, are marked in the plot for water levels to be better observable. It can be seen that suitable sampling intervals would be around or less than 1 second for system to observe all the dynamics. But, since higher frequencies dynamics are not relevant for this work, higher sampling intervals are preferred. Values for the sampling interval of 2 and 5 seconds were used based on previous attempts within other projects. During this work, became clear that 2 seconds is the best option.

In order to test the canal for different operating conditions, an experiment for observing non-linearities and changes on the gain in a gate/pool system was performed, that is demonstrated in figure 6. Only gate 1 was manipulated, leaving all the others steady in such a way that, with the gate almost closed there exists a gap of 730mm in the level from the first pool to the next. Afterwards, gate 1 was progressively opened and the equilibrium of the water level on pool 1 was marked in the graphic, for each situation. The intake flow was also kept constant. Thus, as water pressure is higher on the first pool relatively to the second, small variations on gate 1 will induce big
Figure 5: Behavior of the water surface on a single pool through time and longitudinal axis. Reproduced from [10].

Figure 6: Graphical representation of water level operating points for several gate heights. It was only used gate 1.
changes in the water level of pool 1 since pressure pushes the drainage. Therefore, as the water level equilibrium in pool 1 and the next are approximated, pressure difference between pools is decreased and this is reflected on the effects that manipulation of gate 1 has on pool 1. This means that static gain will be bigger for larger gaps in levels of consecutive pools.

Figure 6 reproduces schematically this situation. Exciting gate 1 at higher levels with a signal $u_1$ leads to a bigger gain than with an input signal $u_2$ at lower levels.

The frequency response of the models identified for higher and lower level situations, presented in figure 7, also confirm this fact. Here, two bode plot are shown for the two identified state-space models with only gate 1 as a manipulated variable. Output variable is thus pool 1. One model is identified within a steady water level of 750mm, being that it corresponds to the higher level model and other is identified within a steady water level of 400mm corresponding to the lower one. As can be observed, static gain in higher level situations is bigger than in lower level situations.

Furthermore, figure 7, also carries information about how quick each system is. Observing this figure, it can be seen that each model has a resonance peak approximately at $6 \times 10^{-1}$. For higher levels this peak has less amplitude than for lower ones. Given this fact, it is possible to say that lower levels have quicker dynamics than higher ones. Carrying this idea to a physical point of view, lower levels will achieve an equilibrium state faster due to larger gate openings which allow more water to drain.

Regarding to the high frequency peak in both situations (figure 7), in this kind of systems it is common. Studies made through Saint-Venant equations [11] confirm the existence of this particular resonant behavior. In figure 8 is shown a bode diagram for only one manipulated gate and one measured water level. All gates were opened except for the last one, so, pool 1, 2 and 3 play the role of a big pool, which originates differences on the bandwidth, relatively to figure 7. Intake flow was manipulated resulting in the family of curves presented in figure 8. As intake flow is decreased, resonant peaks tend to fade away. Analyzing figures 7 and 8 from a qualitative perspective, it can be seen that both have an alike frequency response shape.

### 2.2.1 SISO system identification

For model identification of one-pool/one-gate data pool 3 was chosen. A pseudo random binary signal was applied to the gate around an equilibrium level, as seen in figure 9, for retrieving as much information as possible about the dynamics of the process. For validation the same signal was used but frequency was increased so that desired models can be validated with a different signal.

After the data has been collected from the simulation, firstly, an ARX structure was used to model the data. Using the ARX structure,

$$y(t) + a_1y(t−1) + a_2y(t−2) = b_0u(t−3) + e(t)$$

(8)
Figure 8: Bode plot for one pool and one gate system with several input flows. Gate is of overshoot type. Reproduced from [11].

Figure 9: Open loop response of pool 3 with a flow intake, $Q = 0.02m^3/s$. Gate 1 was kept constant at 34.2mm, gate 2 at 39.6mm and gate 4 at 214.3mm. Gate 3 was manipulated from 31.3mm.
with two poles, one zero and a delay of three samples, results shown on figure 59 are obtained.

Figure 59(b), shows a cross-correlation between input data (gate 3) and residue from output modeling. This is an identification performance analysis and indicates that there are few amount of data from process that are not fitted by the model, since few points go outside the trust region. Figure 59(b) shows the amount of fit covered by the linear model compared to the data. This model presents a 94.8% of fit.

Afterwards, a direct conversion to a state-space model is done. On this conversion the state-space model as a limited shape since many entries in state-space matrices are not used, mainly in the standalone dynamics matrix. So, the model is not properly exploited. Thereby, PEM, is then, used to estimate all entries of these matrices.

Figure 60 shows results concerning to the final state-space model. It can be seen that a state-space conversion followed by a parameter estimation can improve significantly the results, as, by looking to figure 60(a), fitting percentage is increased to 96%. Also, cross correlation between input and residue from output is improved. This happens due to choosing, in first instance, a good starting point for PEM, through state-space representations converted from ARX structures, and so, they present the bottom rung from a quality point of view.

In terms of frequency response only state-space models are discussed. As discussed in [11], open loop frequency response of the transfer function for the last gate, under a certain intake flow, has an appearance similar to figure 8. As intake flow is decreased, resonant peaks at higher frequencies are observed rendering the process to be faster.

Figure 10(a) shows the frequency response of the system identified for SISO system control. It can be seen that such peaks do not exist. This happens due to the fact that low order state-space models are used, which restricts the bandwidth, and so, these resonant peaks are actually neglected. The design of controllers is simplified by using low order models since they already have a clean bandwidth to work on. Another option would be to increase the model order and, then during the controller design, adjust, carefully, the closed loop bandwidth so that any resonant peak is included.

Figure 10(b) illustrates how low order state-space models limit the bandwidth of the process. This figure shows a comparison between the low order model and a higher order model. Both models were identified with the same data, so, in lower frequencies they are much alike. However, as we move on in the frequency axis, the lower order model begins to deviate from the higher order one, constraining the bandwidth. Furthermore, in the phase diagram, it can be seen that the higher order model start to get unstable from certain frequency range, while low order one, does not cross the stability limit. Therefore, more phase margin is available for controller design in the low order model.

2.2.2 MIMO system identification

In the presence of multivariate systems, the same procedure to tackle the identification problem of SISO systems was used. Although, as there are not only one input, it is necessary to combine the effects of each input on each output. To do this, input signals similar to the SISO situation are used, ensuring that step transitions of each gate do not occur at the same instant. So, input signals to consecutive gates were delayed leaving a comfortable time...
Figure 11: MIMO system time response to a step signal, during the identification process applied to all 4 pools. Intake flow was kept constant at $Q = 0.02m^3/s$.

range for stabilizing the water levels in each pool. Offtake action in pools were also taken into account for a further use as accessible disturbances. Offtakes were excited by applying a digital signal similar to the one used on gates. With this additional information, the number of inputs is extended from 4 to 8.

Figure 11 shows the step time response of the overall nonlinear MIMO system when excited with consecutive delayed step signals. Gates 1, 2 and 3 are opened with a $5mm$ amplitude and gate 4 with a $50mm$ amplitude since the last gate has a quite small static gain. Offtakes are not manipulated in this data.

By observing the figure, it can be seen that the behavior of pool 2 and 3 are much alike. Both step responses demonstrate that manipulations performed on a gate located behind pools 2 or 3 could be modeled by 2nd or 3rd order systems since a 2nd order system behavior can be addressed to such systems. This is caused by the flow variation that feeds these pools and comes from previous ones. For manipulations on gates located after pools 2 or 3, a 1st order system behavior is obtained. However pool 1 and 4 do not have the similar behavior of 2 and 3, but both remind a 1st order system. In pool 1, this is explained through the fact it is located at the head of the chain, so, the intake flow that feeds the pool is constant. Pool 4, by the other hand, is directly controlled by gate 4 that has a different outflow structure from the rest, an overshoot drainage. This also reflects itself on the static gain, as can be seen through the figure.

Analyzing these time responses gives the information that lower order systems can be used to identify the data and on how much zeros and poles should be used to model each input/output pair. In fact, lower order models are used in the identification process which yield good results.
For the identification process it is chosen, again, a two stage identification, as in section 2.2.1. In the multivariable case, ARX models have a $N$ input to $M$ output structure, depending on the number of controlled gates plus the number of offtakes ($N$) and the number of pools ($M$). Given this, ARX equation for the first phase model has the next shape,

$$y(t) + A_1 y(t - 1) + A_2 y(t - 2) + ... + A_{na} = B_0 u(t) + B_1 u(t - 1) + B_2 u(t - 2) + ... + B_{nb} u(t - nb - nk) + e(t)$$

where $A_{na}$ is a $[M \times M]$ matrix and $B_{nb}$ $[M \times N]$. By converting the ARX model to the SS model, the B matrix from state-space representation can be arranged in the following manner,

$$\begin{align*}
x(k + 1) &= Ax(k) + Bu(k) + \Psi d(k) \\
y(k) &= Cx(k)
\end{align*}$$

in which $B$ is, now, in its reduced form, $[M \times N/2]$ having only the gates modeling information. $\Psi$ presents the rest of the input modeling, the offtakes. Since the offtake action is not instantaneous, its effect is directly sensed in the state part and not on the output of the overall system. Further ahead, the state-space representation (10) is used in the LQG centralized problem.

As can be seen in figures 65 to 68, that exists a complete coverage of the data with the final state-space representation. Residue analysis for offtake modeling are not shown to not unnecessarily increase the amount of results. The coverage of the data by the model can also be observed through the frequency behavior of the system identified. Figure 12 provides a frequency response analysis based on singular value decomposition (SVD), which in turn are a generalization of bode plots for multivariate systems.

In the frequency response plot for this MIMO system, some high frequency resonant peaks are observed due to typical dynamics of this kind of processes, that should be identified but are not so important in terms of control since controllers, actually, reduce the bandwidth. All singular values are lowpass systems with more or less static gain similar to SISO systems presented in section 2.2.1, being that last gate, with overshot drainage, is the one that possesses a lower gain. From a physical point of view, this is explained by the fact that, in the last gate, water easily drains over it.

### 2.2.3 MIMO with neighboring coupling system identification

The class of systems that must be identified for decentralized controller design are the serially connected systems. As shown in figure 6, these can be decomposed in local linear time invariant subsystems $\Sigma_i$, $i = 1, \ldots, N_{\Sigma}$. Each of these subsystems have a manipulated input $u_i$, a accessible disturbance of $off_i$ and a measured output $y_i$. In addition, $\Sigma_i$ interacts with its neighbors, $\Sigma_{i-1}$ and $\Sigma_{i+1}$. It is assumed that this interaction takes only part through the manipulated variables and the accessible disturbances, being that the cross-coupling of states between systems is neglected.
Figure 13: System decomposition.

Figure 14: Conceptual scheme of the approach used for distributed multivariable identification.
To serve the objectives of this work, these kind of systems must be used since the solution for the decentralized linear quadratic problem requires coupling only through inputs and disturbances of each subsystem. Given this, it will be used an identification scheme similar to 14 applied to each subsystem. Therefore, in identification process, subsystems that further ahead are named agents in the cooperation process, are treated as single MISO (multiple-input single-output) systems with 6 inputs to 1 output. For pools located at the beginning and at the end of the canal, their MISO models become 4-by-1 input/output systems since there are no neighbors at their sides. ARX structures are used, firstly, to model such MISO systems.

After having a ARX representation for each MISO system, a conversion into state-space representation is done to each individual ARX structure. This conversion process is the same as done before in sections 2.2.1 and 2.2.2 but applied to each subsystem, so details and results comparing ARX and state-space representations are skipped. For achieving the final model, all systems are, then, combined together as a global MIMO system in the following state equation,

\[
\begin{bmatrix}
    x_1(k+1) \\
    x_2(k+1) \\
    x_3(k+1) \\
    x_4(k+1)
\end{bmatrix} = \begin{bmatrix}
    A_{1,1} & 0 & 0 & 0 \\
    0 & A_{2,2} & 0 & 0 \\
    0 & 0 & A_{3,3} & 0 \\
    0 & 0 & 0 & A_{4,4}
\end{bmatrix} \begin{bmatrix}
    x_1(k) \\
    x_2(k) \\
    x_3(k) \\
    x_4(k)
\end{bmatrix} + \begin{bmatrix}
    B_{1,1} & 0 & 0 & 0 \\
    0 & B_{2,2} & 0 & 0 \\
    0 & 0 & B_{3,3} & 0 \\
    0 & 0 & 0 & B_{4,4}
\end{bmatrix} \begin{bmatrix}
    u_1(k) \\
    u_2(k) \\
    u_3(k) \\
    u_4(k)
\end{bmatrix}
\]

\[
\begin{bmatrix}
    y_1(k) \\
    y_2(k) \\
    y_3(k) \\
    y_4(k)
\end{bmatrix} = \begin{bmatrix}
    C_{1,1} & 0 & 0 & 0 \\
    0 & C_{2,2} & 0 & 0 \\
    0 & 0 & C_{3,3} & 0 \\
    0 & 0 & 0 & C_{4,4}
\end{bmatrix} \begin{bmatrix}
    x_1(k) \\
    x_2(k) \\
    x_3(k) \\
    x_4(k)
\end{bmatrix}
\] (11)

in which \(A_{i,i} [n \times n] \), \(B_{i,i} [n \times 1] \) and \(C_{i,i} [1 \times n] \) correspond to the \(A \), \(B \) and \(C \) matrices used to model subsystem \(i \). \(\Psi_{i,j} [n \times 1] \) and \(\Gamma_{i,j} [n \times 1] \) represent the adjacent subsystem’s modeling with respect to subsystem \(i \) and the offtake modeling information, respectively. Here, \(n \) is the number of states that are used to describe each subsystem that is the same for all of them. Thereby, \(d_{\text{gated}} \) represents the disturbance that arises from movements in gate \(i \) and could well be called \(u_i \) since \(\Psi_{i,i} \) models the input dynamics with respect to adjacent gates, concerning subsystem \(i \). But it will be better to separate it, looking forward to the decentralized control algorithm. \(d_{\text{off}} \) represents disturbances coming from offtake of subsystem \(i \).

System (11) can, also, be formulated, without becoming restrictive,

\[
x_i(k+1) = A_{i,i} x_i(k) + B_{i,i} u_i(k) + \sum_{j=1}^{j=4} \Psi_{i,j} d_{\text{gated}} + \sum_{j=1}^{j=4} \Gamma_{i,j} d_{\text{off}}
\] (12)

\[y_i(k) = C_{i,i} x_i(k)\]

From a control perspective, each local system (12) can be considered as a SISO system with input \(u_i \) and output \(y_i \). Accessible disturbances from offtakes, in this case, are not considered inputs since they are not manipulated by controllers designed. Offtakes are seen as aleatory variables, user manipulated, that simulate unscheduled water discharges.

For previous state-space representation, it is clear that this choice reduces effectiveness in identification, more than a complete coupled model would that could add standalone dynamics. Thus matrix \(A \) would have a tri-diagonal structure, instead of a block diagonal structure.

By looking at the residue analysis presented in figures 69 to 72 comparisons can be made with results for centralized approach. This decentralized structures identified can not describe the data from the process just as well as
centralized ones. This is explained by the chained effects that gates far away have in levels along the canal. Where this fact is more noticed is mostly for the last pools, that gather all water flow coming from previous gates that are far away and are not taken into account.

In terms of frequency, makes no sense to analyze this case since coordination between agents is an iterative process framed between two consecutive sampling time instants, and so, it is impossible to model an online iterative algorithm.

3 Optimal Control Problem

In this section, the LQ Control theory for discrete-time will be discussed to tackle the decentralized LQG controller problem with offtake feedforward control for canal systems and improvements on LQ control through constrained closed loop poles. A reference to Kalman-Bucy filter theory will also be made.

Let,
\[ \dot{x} = f(x(t), u(t)) \quad ; \quad t \in [0, T] \] (13)
describe the dynamics of a given system for control proposes, with a state \( x(t) \) and a control input \( u(t) \). Optimal Control is the answer for problems related with system actuation when energy minimization is a target objective and can be formulated by,
\[ \bar{J}(u) = \Psi(x(T)) + \int_0^T L(x, u) \, dt ; \quad u(t) \in U \] (14)

It stands for finding a function \( u \) that minimizes the cost function \( J(u) \) in a bounded time interval. So \( J(u) \) transforms each element from the space of possible functions into a real number. To solve this type of problems Pontryagin Principle must be applied since typical optimization methods do not minimize a cost functional through a set of functions.

One particularization of Optimal Control problems are Linear Quadratic problems, in which quadratic cost functions are associated such that a linear plant is to be controlled. This kind of problems are the startup for this work. The Linear Quadratic problem within an infinite horizon approach for simplifying the Ricatti solution obtained in this kind of problems, being that, it remains constant throughout all control range. So the general cost function used is described by,
\[ J_\infty = \frac{1}{2} \int_0^\infty [x^T(t)Qx(t) + u^T(t)Ru(t)] \, dt \] (15)

\[ Q \geq 0 \quad R > 0 \]

3.1 Decentralized LQ control with accessible disturbances

Decentralized controllers were designed only for 4 input/4 output MIMO systems, the general case for the given problem. Other controllers for particular situations, such as 1 gate/1 pool, 2 gates/2 pools and 3 gates/3 pools were addressed only with the LQG centralized approach.

The system for which decentralized controller design is to be applied was already discussed in section (2.2.3). Only to contextualize, a MIMO system describes the dynamics of the overall process which can be decomposed in several subsystems, SISO systems. These subsystems are authorized to communicate and change information about their neighbor’s strategy for control. Standalone dynamics that account for the influence that the outputs have on each other are not considered.

The decentralized control problem, discussed in this work, consists of attaching a local controller to each SISO system, that is, each local controller is allowed only to manipulate its respective subsystem. Figure 15 presents a
representative scheme of the control problem at hand, where, $d_i$ represents the disturbance from offtake $i$, $u_i$ the control input to the subsystem $i$ and $y_i$ the water level obtained from subsystem $i$. Local controllers are represented by $C_i$ and subsystems by pool $i$. Although $d_i$ is attached to output $y_i$ on the communication from pool $i$ to controller $C_i$, these signals do not represent system outputs because manipulation of $u_i$ has no effect on $d_i$. Offtakes are manipulated by user request and not by control action.

As controllers only have a local impact in the control of the overall MIMO system, a coordination strategy between them is, thus, needed to achieve a shared solution. For this purpose, information is exchanged between the controllers about each one’s strategy, which makes a serial connection throughout all of them. Offtake action is, also, transmitted through this connections in a decentralized manner, that is, subsystem $i$ only has access to the offtake signal from subsystems $i-1$ and $i+1$.

For the controller design it is chosen a LQ regulator based controller. The performance index with which the LQ problem is to be solved is,

$$ J = \frac{1}{2} \sum_{t=1}^{\infty} [y^T(k)y(k) + u^T(k)Ru(k)] dt \quad ; \quad R > 0 $$

(16)

where $y(k)$ denotes the output of the process. This is a particular case of the performance index (15) in which matrix $Q$ is chosen by making $Q = C^T C$.

The assumptions with respect to the system used for control and for the performance index allow the decentralized control design based on the LQ solution within an infinite horizon, that is presented in appendix A.

In the control law solution, there are 3 types of control inputs. Each controller has a state feedback control input, related with the solution from an algebraic Riccati equation, that is only responsible for stabilizing its water level; an iterative control algorithm that converges to a coordination between neighboring controllers, being that, each controller sees its neighbor’s control strategy; and a feedforward control that uses offtake disturbances coming from neighbors as well.

### 3.1.1 Local state feedback control

For the controller to follow a given reference signal, that is, a desired water level, reference accommodation is needed. Reference accommodation by integral effect was chosen. This makes the choice of suitable gains simpler because the integrator error can be, directly, fed into the LQ problem by augmenting the state equation that describes the process.

In order to use integral effect of the tracking error, a difference state equation relatively to the reference accommodation is created,

$$ x_I(k+1) - x_I(k) = e(k) \iff x_I(k+1) = x_I(k) + (r(k) - y(k)) \iff (17) $$

$$ \iff x_I(k+1) = x_I(k) + (r(k) - Cx(k)) $$

where $x_I$ is the state of the error integrator, $r(k)$ the desired reference, $y(k)$ the process output and $e(k)$ the error between the reference desired, $r(k)$, and the output, $y(k)$. Therefore, state equation (11) that describes the process is augmented.
\[
\begin{bmatrix}
x(k+1) \\
x_I(k+1)
\end{bmatrix} =
\begin{bmatrix}
A & 0 \\
-C & 1
\end{bmatrix}
\begin{bmatrix}
x(k) \\
x_I(k)
\end{bmatrix} +
\begin{bmatrix}
B \\
0
\end{bmatrix} u(k) +
\begin{bmatrix}
\Psi \\
0
\end{bmatrix} d_{\text{gate}}(k) +
\begin{bmatrix}
\Gamma \\
0
\end{bmatrix} d_{\text{off}}(k) +
\begin{bmatrix}
0 \\
1
\end{bmatrix} r(k)
\]
(18)

\[
y(k) =
\begin{bmatrix}
C & 0
\end{bmatrix}
\begin{bmatrix}
x(k) \\
x_I(k)
\end{bmatrix}
\]

However, for the overall Riccati solution to appear decentralized, one Riccati Algebraic discrete-time equation must be solved for each agent and not to the global MIMO system. This means that the system (18) needs to be rearranged, in such a way that the overall Riccati solution do not mix all states in the control law. Even though there is not a centralized model available for the process, the direct optimization of the LQ loss function (16) will always mix the agents states.

To compute the Riccati solution for each agent, let the system (18) be simplified as follows,

\[
\begin{bmatrix}
x_i(k+1) \\
x_{Ii}(k+1)
\end{bmatrix} =
\begin{bmatrix}
A_{i,i} & 0 \\
-C_{i,i} & 1
\end{bmatrix}
\begin{bmatrix}
x_i(k) \\
x_{Ii}(k)
\end{bmatrix} +
\begin{bmatrix}
B_{i,i} \\
0
\end{bmatrix} u_i(k)
\]
(19)

\[
y_i(k) =
\begin{bmatrix}
C_{i,i} & 0
\end{bmatrix}
\begin{bmatrix}
x_i(k) \\
x_{Ii}(k)
\end{bmatrix}
\]
or in a compacted form,

\[
\dot{x}_i(k+1) = \bar{\lambda}_{i,i} \dot{x}_i(k) + \bar{B}_{i,i} \dot{u}_i(k)
\]
\[
\dot{y}_i(k) = \bar{C}_{i,i} \dot{x}_i(k)
\]

where the index \( i \) represents the local scenario applied to agent \( i \). \( d_{\text{gate}} \) and \( d_{\text{off}} \) disappear from equation (18) because the Riccati solution does not depend on these, as demonstrated in appendix A.

The simplified system (20) can, then, be attached to the performance index (16) for the decentralized LQ control design of each system.

For the state feedback based control part, four Riccati equations are solved, one for each system (19),

\[
P^i = \bar{A}_{i,i}^T P^i[I + \frac{1}{\rho_i} \bar{B}_{i,i} \bar{B}_{i,i}^T P^i]^{-1} \bar{A}_{i,i} + \bar{C}_{i,i}^T \bar{C}_{i,i}
\]
(21)

where \( P^i \) is the Riccati solution for system \( i \) and \( \rho_i \) the cost associated with the energy minimization for the control input to system \( i \). Thus, \( P^i \) is described by,

\[
P^i =
\begin{bmatrix}
P^i_{\dot{x},x} & P^i_{\dot{x},\lambda_i} \\
P^i_{\dot{\lambda}_i,x} & P^i_{\dot{\lambda}_i,\lambda_i}
\end{bmatrix}
\]
(22)

so that the co-state solution is as follows,

\[
\begin{bmatrix}
\dot{\lambda}_i(k) \\
\dot{\lambda}_{Ii}(k)
\end{bmatrix} =
\begin{bmatrix}
P^i_{\dot{x},x} & P^i_{\dot{x},\lambda_i} \\
P^i_{\dot{\lambda}_i,x} & P^i_{\dot{\lambda}_i,\lambda_i}
\end{bmatrix}
\begin{bmatrix}
x_i(k) \\
x_{Ii}(k)
\end{bmatrix}
\]
(23)

in which \( x_i(k) \) \([n \times 1]\) is defined as the set of states of system \( i \) and \( \lambda_i \) its co-state solution. \( x_{Ii}(k) \) \([1 \times 1]\) refers to the integral error state of output \( i \) and \( \lambda_{Ii} \) its co-state solution. \( P^i_{\dot{x},x} \) \([n \times n]\) defines the Riccati solution part that accounts for the component of \( x \) on \( \lambda \) for agent \( i \), \( P^i_{\dot{x},\lambda_i} \) \([n \times n]\) the vector that represents the component of \( x_I \) on \( \lambda \), \( P^i_{\dot{\lambda}_i,x} \) \([1 \times n]\) the transposed vector that accounts for the component of \( x \) on \( \lambda_I \) for agent \( i \) and \( P^i_{\dot{\lambda}_i,\lambda_i} \) that is a scalar and accounts the component of \( x_I \) on \( \lambda_I \). In addition, \( n \) is the number of states that each agent possesses, assumed to be equal for all.
Now, as a general format of $P$ is used in computations, system states and effect integral states must be rearranged and separated. Computations for each system could also be carried out and there would be no reason to have a general matrix $P$, but in this case, as this is possible to do, a matrix calculus based computation is preferred. Yet, in a real decentralized controller configuration, this could not be done.

The general matrix $P$ is created,

$$
P = \begin{bmatrix}
P_{1,1} & 0 & 0 & P_{1,3}^1 & 0 & 0 & 0 \\
0 & P_{2,1} & 0 & 0 & P_{2,2}^1 & 0 & 0 \\
0 & 0 & P_{3,1} & 0 & 0 & P_{3,2}^1 & 0 \\
0 & 0 & 0 & P_{4,1} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & P_{4,2}^1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & P_{4,3}^1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & P_{4,4}^1
\end{bmatrix}
$$

(24)

so that the co-state is arranged as follows,

$$
\begin{bmatrix}
\lambda(k) \\
\zeta(k) \\
x_I(k)
\end{bmatrix} = P \begin{bmatrix}
x(k) \\
x_I(k)
\end{bmatrix}
$$

(25)

where systems states $x(k) \ [4n \times 1]$ and the effect integral states $x_I(k) \ [4 \times 1]$ are, in fact, separated.

Furthermore, the overall optimal control law, as described in appendix A is as follows,

$$
u_{opt}(k) = -K_{fb}x(k) - K_Ix_I(k) + u_{gate}(k) + u_{off}(k)
$$

(26)

where $K_{fb}$, again, represents the optimal control solution for the dynamics states feedback and $K_I$ is introduced due to the optimal control solution as a result of the error integration. Following the optimal control law computations presented in appendix A, $K_I$ and $K_{fb}$, have the matrix structures desired,

$$
K_{fb} = \begin{bmatrix}
K_{fb_1} & 0 & 0 & 0 \\
0 & K_{fb_2} & 0 & 0 \\
0 & 0 & K_{fb_3} & 0 \\
0 & 0 & 0 & K_{fb_4}
\end{bmatrix}
$$

(27)

$$
K_I = \begin{bmatrix}
K_{I_1} & 0 & 0 & 0 \\
0 & K_{I_2} & 0 & 0 \\
0 & 0 & K_{I_3} & 0 \\
0 & 0 & 0 & K_{I_4}
\end{bmatrix}
$$

(28)

where both $K_{fb} \ [1 \times n]$ and $K_I \ [1 \times 1]$, only use their $i$ states for computing feedback control for agent $i$.

$u_{gate}$ and $u_{off}$ account for the optimal control solution to the coordination between agents and to the offsets action, that are addressed in subsections 3.1.2 and 3.1.3, respectively. The general matrix $P$ (24) is also used for computing $u_{gate}$ and $u_{off}$.

### 3.1.2 Coordination between controllers

Now, focusing on the coordination part of the equation (26), consider an agent $i$ with control law $u_{gate_i}$ and its neighbors $i-1$ and $i+1$ that also have control laws $u_{gate_{i-1}}$ and $u_{gate_{i+1}}$. Agent $i$ receives information on the manipulated variables from the adjacent agents and changes its manipulated variable attending to the neighbors strategy. On the other hand, other agents, $i-1$ and $i+1$, also receive manipulated variable information of agent $i$ and change their control strategy. Thus the coordination algorithm must be iterated until all agents in the network achieve a consensus.

The coordination game is done at the beginning of each sampling interval, for the agents to agree on what will be
the next control value in which everyone benefits. This means that \( u_{\text{gate}}(k, \tau) \) not only depends on the sampling interval instant but also changes along the iteration index \( \tau \), as agents change information with each other. Figure 16 illustrates the coordination within two sampling intervals, \( k \) and \( k + 1 \). Assuming that \( \tau \) is the iteration index, the algorithm can be synthesized as follow,

1. Set the index, \( \tau = 1 \).

2. Within two sampling intervals, \( k \) and \( k + 1 \), update controller \( i, i = 1, 2, 3, 4 \) with its neighbors \( i - 1 \) and \( i + 1 \) by,

   \[
   u_{\text{gate}_i}(k, \tau + 1) = \mathcal{F}(u_{\text{gate}_{i-1}}(k, \tau), u_{\text{gate}_{i+1}}(k, \tau))
   \]  
   \[(29)\]

   where \( \mathcal{F}(\cdot) \) denotes the procedure used for the coordination that in this problem is a linear vector function.

3. If convergence is reached, stop and apply control law to the plant. Otherwise, move on to the next iteration \( (\tau = \tau + 1) \) and go back to step 2.

Nevertheless \( \mathcal{F}(\cdot) \) needs to be found. Since it is a linear vector function it can be decomposed in,

\[
\begin{bmatrix}
    u_{\text{gate}_1}(k, \tau + 1) \\
    u_{\text{gate}_2}(k, \tau + 1) \\
    u_{\text{gate}_3}(k, \tau + 1) \\
    u_{\text{gate}_4}(k, \tau + 1)
\end{bmatrix} = M_{\text{coord}}
\begin{bmatrix}
    u_{\text{gate}_1}(k, \tau) \\
    u_{\text{gate}_2}(k, \tau) \\
    u_{\text{gate}_3}(k, \tau) \\
    u_{\text{gate}_4}(k, \tau)
\end{bmatrix}
\]  
\[(30)\]

In order to find \( M_{\text{coord}} \), equation (65) and equation (75), picking \( g_{\text{gate}} \), are combined and \( \bar{g}_{\text{gate}} \) is introduced, in such a way that \( u_{\text{gate}} \) becomes,

\[
u_{\text{gate}}(k, \tau + 1) = (I + R^{-1}B^T PB)^{-1}R^{-1}B^T (\bar{g}_{\text{gate}} - P\Psi) u_{\text{gate}}(k, \tau)
\]  
\[(31)\]

where \( \bar{g}_{\text{gate}} \) is obtained by just pushing \( u_{\text{gate}}(k, \tau) \) out of the parentheses, in the equation for \( g_{\text{gate}} \). Therefore, keeping in mind that the overall solution is decentralized, that is, calculations are locally performed, equation (31) can be simplified to,

\[
u_{\text{gate}}(k, \tau + 1) = 
\begin{bmatrix}
    M_{\text{coord}_{1,1}} & 0 & 0 & 0 \\
    0 & M_{\text{coord}_{2,2}} & 0 & 0 \\
    0 & 0 & M_{\text{coord}_{3,3}} & 0 \\
    0 & 0 & 0 & M_{\text{coord}_{4,4}}
\end{bmatrix}
\begin{bmatrix}
    u_{\text{gate}_1}(k, \tau) \\
    u_{\text{gate}_2}(k, \tau) \\
    u_{\text{gate}_3}(k, \tau) \\
    u_{\text{gate}_4}(k, \tau)
\end{bmatrix}
\]  
\[(32)\]
where $M_{\text{coord}, i}$ is a scalar and represents the decentralized gain that agent $i$ uses to achieve a consensus with the neighboring agent $j$, and is fixed throughout all simulation.

### 3.1.3 Feedforward control with offtakes

The feedforward control action from accessible disturbances is similar to the one used in subsection 3.1.2. Combining equation (66) with equation (75), picking $g_{\text{off}} - F_{\text{FF}}$, yields,

$$u_{\text{off}}(k) = M_{\text{FF}} d_{\text{off}}(k) \quad (33)$$

where $M_{\text{FF}}$ is the tri-diagonal constant matrix,

$$M_{\text{FF}} = (I + R^{-1}B^T P B)^{-1}R^{-1}B^T (\bar{g}_{\text{off}} - P \Gamma) \quad (34)$$

that does a weighting of each neighbor’s disturbance for feedforward control. Here $\bar{g}_{\text{off}}$ plays the same role as $\bar{g}_{\text{gate}}$ in equation (31).

### 3.2 Coordination Convergence Criteria

The algorithm presented in section 3.1 for agent coordination is an iterative algorithm, and so, at a glance, there is no guarantee that it will converge. Hereafter, a convergence condition for the distributed coordination is presented.

For finding a convergence criterion, consider equation (30) that defines a consensus between different agents. It can be seen that it is similar to a simple state-space system with standalone dynamics, $x(k + 1) = Ax(k)$. Analyzing it from a stability perspective, it can be shown that the state-space solution will be stable if all eigenvalues of $A$ are stable, meaning that it will reach an equilibrium when $k \to \infty$. This result is achieved by expanding the discrete-time solution and, therefore, reaching the form that the solution is driven by terms of $\lambda_k$, where $\lambda_k$ are the eigenvalues of $A$. Calculations that lead to the solution for continuous-time case are presented in [13] and the same method can be applied to the discrete-time situation.

So, for equation (30) to converge the criterion adopted must be,

$$\lambda := \max |\text{eig}(M_{\text{coord}})| < 1 \quad (35)$$

In figure 17 a diagonal matrix was initially defined by,

$$R = \begin{bmatrix} 20 & 10 & 2 & 1 \end{bmatrix}$$
that represents the fixed LQ weights used in the simulation performed. These weights were, then, multiplied by a coefficient $\rho$ with logarithmic spanned values, to show the influence of $R$ on the convergence in distributed coordination. The maximum eigenvalue is, then, picked in each LQ weights set. In the figure a limit line for $\lambda = 1$ is drawn that corresponds to the marginal situation when coordination fails to converge.

It can be seen that for low values of the weights the algorithm does not converge but for higher values it does converge. This is explained due to the fact that with lower weights, the response of LQ control becomes fast and more demanding. As the weights increase, the energy usage by each agent in control is constrained due to the LQ performance index, so the response will, thereby, be smoothed which improves the coordination between agents.

3.3 Constraining Closed Loop Poles

In this section an improvement in LQ control is presented. A disadvantage of LQ control is that its response sometimes is oscillatory with a big overshoot value that is not desired. Pole constraining in the closed loop case is, then, introduced to address this situation.

This problem was originally formulated in continuous-time [14]. Hereafter the corresponding solution in discrete-time is presented.

What constraining closed loop poles, in the actual problem, does is to push and restrict poles, under restrictions already imposed by the LQ optimal solution, to a smaller radius, thereby approximating poles from origin. And, as poles move towards the origin, closed loop system tend to be less oscillatory since they are moving away from the marginally stable region, $|z| = 1$.

Consider the system already presented in equation (11).
To force closed loop poles to lay in a circle, the performance index (16) is changed to,

$$\tilde{J} = \sum_{t=1}^{\infty} \left[ x^T(k)C^T Cx(k) + u^T(k)Ru(k) \right] \alpha^{2k}$$

(36)

where the radius of the circle is given by $1/\alpha$ and $\alpha > 1$.

In order to minimize $J$, a change in the variables, $x(k)$ and $u(k)$, is done,

$$\tilde{x} = x\alpha^k \quad \tilde{u} = u\alpha^k$$

(37)

or

$$x = \tilde{x}\alpha^{-k} \quad u = \tilde{u}\alpha^{-k}$$

(38)

Replacing (38) in (11) yields

$$\tilde{x}(k+1)\alpha^{-k} = A\tilde{x}(k)\alpha^{-k} + B\tilde{u}(k)\alpha^{-k} + \Psi d_{gate}(k) + \Gamma d_{off}(k)$$

or

$$\tilde{x}(k+1) = \alpha A\tilde{x}(k) + \alpha B\tilde{u}(k) + \Psi d_{gate}(k)\alpha^{k+1} + \Gamma d_{off}(k)\alpha^{k+1}$$

(39)

and $\tilde{J}$ becomes

$$\tilde{J} = \sum_{k=1}^{\infty} \left[ \tilde{x}^T(k)C^T C\tilde{x}(k) + \tilde{u}^T(k)R\tilde{u}(k) \right]$$

(40)

Thus, minimizing (36) with dynamics (11) is the same as minimizing (40) with dynamics (39). The solution of minimizing (40) with dynamics (39) is easily obtained by replacing

$$A \rightarrow \alpha A \quad B \rightarrow \alpha B$$

(41)

and applying the standard LQ design formulas to the dynamics (39).
3.4 DLQ example: Coordination between two double-integrator plants

The following example uses two double-integrator plants, $\Sigma_1$ and $\Sigma_2$, each one described through an independent state-space realization. Both plants add, to their own dynamics, the disturbance that comes from its adjacent plant, $u_2$ and $u_1$, respectively. Nevertheless, a possible coupling through state information of both plants is absent in this example.

Figure 18(a) shows a schematic representation of the two plants and their interconnections. Consider that the plants are described by,

$$x_1(k+1) = \begin{bmatrix} 1 & h \\ 0 & 1 \end{bmatrix} x_1(k) + \begin{bmatrix} \frac{h^2}{2} \\ \frac{h}{2} \end{bmatrix} u_1(k) + \begin{bmatrix} \frac{h^2}{2} \\ \frac{h}{2} \end{bmatrix} u_2(k)$$

(42)

$$y_1(k) = \begin{bmatrix} 1 & 0 \end{bmatrix} x_1(k)$$

for plant $\Sigma_1$ and

$$x_2(k+1) = \begin{bmatrix} 1 & h \\ 0 & 1 \end{bmatrix} x_2(k) + \begin{bmatrix} \frac{h^2}{2} \\ \frac{h}{2} \end{bmatrix} u_2(k) + \begin{bmatrix} \frac{h^2}{2} \\ \frac{h}{2} \end{bmatrix} u_1(k)$$

(43)

$$y_2(k) = \begin{bmatrix} 1 & 0 \end{bmatrix} x_2(k)$$

for plant $\Sigma_2$.

Two separate LQ controllers are designed, one for each plant. LQ controller $K_1$ that controls the plant $\Sigma_1$ takes the accessible disturbance $u_2$ into account for control and $K_2$ responsible for the plant $\Sigma_2$, also takes the accessible disturbance $u_1$ for control. This makes the decentralized solution to be calculated recursively, as in section 3.1. Figure 18(b) illustrates the formulation of the problem.

It was necessary to use the improvement for LQ controllers referred in section 3.3 to stabilize the decentralized LQ controllers, using $\alpha = 1.02$.

Figures 19 and 20 show the time response of the plants presented. Let $N_c$ be the number of iterations that are done with the distributed coordination scheme, after each step transition in the desired reference. With a small $N_c$ controllers are unable to stabilize the plants. This is due to the fact that with a finite number of iterations, solution achieved is a sub-optimal one, and agents need to go even further in the negotiation process. Only for $N_c = 10$, as can be seen in figure 19, controllers manage to stabilize $\Sigma_1$ with a much oscillatory time response but $\Sigma_2$ still appears to be close to a marginally stable situation. Increasing $N_c$ leads to improvements in results, as seen in figure 20. $N_c = 80$ is used. But these improvements do not grow linearly with the choice of $N_c$. They have a decreasing exponential behavior through $N_c$. This is observed in figure 21, as it is shown the loss function obtained from the mean square error of the tracking error as a function of the number of iterations, $N_c$. Through the figure is observed that for lower values of $N_c$, large improvements on results are noted but from a certain number of iterations, they do not significantly improve.
Figure 19: DLQ example. Time response of the two double-integrator plants with coordination control. $N_c = 10$, $\alpha = 1.02$.
Figure 20: DLQ example. Time response of the two double-integrator plants with coordination control. $N_c = 80, \alpha = 1.02$.

Figure 21: DLQ example. Loss as a function of the number $N_c$ of iterations in the coordination step.
Figure 22: DLQ example. Manipulated variables as a function of number of iterations \( (N_c) \) in the coordination step for \( t = 350 \).

Figure 22 shows the two manipulated variables of the plants through their iterative process of coordination within a reference step transition at \( t = 350 \). Both manipulated variables reach an equilibrium for about 150 iterations.

### 3.5 Steady-State Kalman Filtering

The usage of LQ controllers in nonlinear processes implies the estimation of the system state at every sampling interval. Since in the case considered, the system state is not accessible, an estimation for the state through an observer should be provided.

A Kalman-Bucy estimator is used in this work due to the optimal properties it possesses with respect to noise and disturbance filtering, when their statistical behavior is known. Therefore, this section discusses the general steps to reach the kalman filter solution.

In the centralized control approach, standard kalman filter theory together with Loop Transfer Recovery (LTR) [15] is applied to the 1 gate/1 pool centralized controller, where minimal phase models can be found. When dealing with MIMO systems, finding minimal phase models is not an easy task since zero analysis, in this case, is not a trivial subject. Then, the recovery of the LQ properties with respect to the gain margin and stability by filtering noise within the process can no longer be ensured.

Although it is possible to perform an analysis to the open-loop zeros of the system identified, that is out of the scope of this work.

For decentralized control a decentralized observer is designed so that a decentralized compensator may be tested on the experimental canal, which represents the purpose of the work.

This section focus only the decentralized approach, since the centralized one provides no academic contribution.

Consider the system (11) but without any kind of coupling, that is, matrix \( \Psi \) is ignored, \( C \) is approximated to a block diagonal matrix by removing off-diagonal elements and \( \Gamma \), that plays a disturbance role in the model, is,
also, not taken into account in the state estimation. Thus, the system seen by the observer has the structure,

\[
\begin{bmatrix}
    x_1(k+1) \\
    x_2(k+1) \\
    x_3(k+1) \\
    x_4(k+1)
\end{bmatrix} =
\begin{bmatrix}
    A_{1,1} & 0 & 0 & 0 \\
    0 & A_{2,2} & 0 & 0 \\
    0 & 0 & A_{3,3} & 0 \\
    0 & 0 & 0 & A_{4,4}
\end{bmatrix}
\begin{bmatrix}
    x_1(k) \\
    x_2(k) \\
    x_3(k) \\
    x_4(k)
\end{bmatrix} +
\begin{bmatrix}
    B_{1,1} & 0 & 0 & 0 \\
    0 & B_{2,2} & 0 & 0 \\
    0 & 0 & B_{3,3} & 0 \\
    0 & 0 & 0 & B_{4,4}
\end{bmatrix}
\begin{bmatrix}
    u_1(k) \\
    u_2(k) \\
    u_3(k) \\
    u_4(k)
\end{bmatrix} +
\begin{bmatrix}
    w_1(k) \\
    w_2(k) \\
    w_3(k) \\
    w_4(k)
\end{bmatrix}
\] (44)

\[
\begin{bmatrix}
    y_1(k) \\
    y_2(k) \\
    y_3(k) \\
    y_4(k)
\end{bmatrix} =
\begin{bmatrix}
    C_{1,1} & 0 & 0 & 0 \\
    0 & C_{2,2} & 0 & 0 \\
    0 & 0 & C_{3,3} & 0 \\
    0 & 0 & 0 & C_{4,4}
\end{bmatrix}
\begin{bmatrix}
    x_1(k) \\
    x_2(k) \\
    x_3(k) \\
    x_4(k)
\end{bmatrix} +
\begin{bmatrix}
    v_1(k) \\
    v_2(k) \\
    v_3(k) \\
    v_4(k)
\end{bmatrix}
\]

where \( A_{ij} [n \times n] \), \( B [n \times 1] \) and \( C [1 \times n] \) represent the individual model structure for agent \( i \) and \( w_i(k) [4n \times 1] \) and \( v_i(k) [4 \times 1] \) are white noise signals that affect agent \( i \). This approximation, mainly by neglecting the coupling through \( C \) and \( \Psi \), induces modeling errors into the observer state but as it is shown further ahead, it produces good results.

Kalman filter is described through the observer equation,

\[
\hat{x}(k+1) = A\hat{x}(k) + Bu(k) + M(y(k) - C\hat{x}(k))
\] (45)

where \( \hat{x}(k) \) is the estimate of the states and \( M \) is the observer gain,

\[
M = \begin{bmatrix}
    M_{11} & 0 & 0 & 0 \\
    0 & M_{22} & 0 & 0 \\
    0 & 0 & M_{33} & 0 \\
    0 & 0 & 0 & M_{44}
\end{bmatrix}
\] (46)

computed from the four Riccati Algebraic discrete-time equation, as in [16], one for each agent,

\[
S_i = A_i S_i A_i^T - (A_i S_i C_i^T) (C_i S_i C_i^T + R)^{-1} (C_i S_i A_i^T) + Q_0
\] (47)

\[
M_{ii} = S_i C_i^T (C_i S_i C_i^T + R)^{-1}
\] (48)

in which \( S_i \) represents the solution for the Riccati equation corresponding to agent \( i \).

Kalman-Bucy filters require the knowledge the statistical properties of noise. As these filters optimize the bandwidth tradeoff between model dynamics and noise power of the controller loop gain, the hypothetical noise and disturbance covariances are delivered to the optimization problem. They assume the statistical properties of the noise as a white noise source,

\[
E \{ w(k)w^T(k + \tau) \} = Q_0 \delta(\tau) \quad E \{ v(k)v^T(k + \tau) \} = R_0 \delta(\tau)
\] (49)

where \( Q_0 \) and \( R_0 \) are the noise covariances. A common design practice consists in estimating \( Q_0 \) and \( R_0 \), using these as tuning knobs that are adjusted so that the loop gain has the desired relative stability margins. Loop Transfer Recovery (LTR) [15] provides a method for such purpose.

As LTR theory conditions are not strictly satisfied for the multivariable cases considered here, the good relative stability margins that LQ controllers with full-state feedback possess, can no longer be guaranteed. Then, noise and disturbances parameters need to be manipulated in such a way that the LQ full-state loop gain including the observer has the desired properties. This is transversal to all multivariable cases presented in this work, in centralized and decentralized control design.

Noise covariances were, then, adjusted by increasing the parameters, \( Q_0 \) and \( R_0 \), so that Kalman observer is not mistaken by modeling errors included in the model. However, LQ controllers performance gets worse as these parameters are increased since Kalman observer tend to reduce the open loop gain bandwidth, which renders the closed loop dynamics to be slower.

Therefore, a manual setting of these parameters must be done when designing the Kalman observer.
4 Centralized Control Design

In this section, centralized versions of the LQ controller introduced in section 3.1 are presented. These versions are designed for the overall full coupled MIMO system discussed in section 2.2.2 and, also, for the SISO system discussed in section 2.2.1. Results on these are shown, both for the simulation model, throughout this section and for the experimental canal, in the section REF(CANAL EXPERIMENTAL). As feedforward control through offtakes is used in decentralized control, a centralized version for it is designed for MIMO systems and discussed in section 4.4.

Although centralized controllers are important for comparing their performance with decentralized ones, details in the design of centralized controllers are out of the scope of this work and, so, they are not generically discussed in this work. A centralized LQG controller and uncoordinated LQG controllers are presented in sections 4.1 and 4.2, respectively, being that the usage of both have advantages and disadvantages and they are useful to compare their performances with the decentralized control case. These controllers do not include offtake feedforward control. Further ahead, a section regarding the results obtained for controllers with the pole radius constraint improvement is included in the section, covering also in what way this affects the controllers performance. Finally, simplifications in control through model order reduction and through controller order reduction are presented as well as a comparison between both with the full order case, in terms of performance.

For the design of centralized controllers the LQ regulator performance index (16) is used, together with the full coupled state dynamics (10) or with the SISO system representation, corresponding to the MIMO or SISO controller design. A slight change is done in these systems, as in the decentralized control case in section 3.1, in order to include the reference with integral effect. This means that the number of states is to be increased and the state dynamics matrix \( A \) needs to be augmented before attaching the system to the performance index, as done in section 3.1.1. To solve the optimization problem, the standard equations for LQ regulator design in discrete-time are used, thus achieving the typical centralized solution for LQ regulators.

The centralized control law is then

\[
    u_{opt}(k) = -K_{fb}x(k) - K_Ix_I(k)
\]

where \( K_{fb} \) \([n \times 4]\) and \( K_I\) \([4 \times 4]\) are the full-state optimal control gains to be applied on the nonlinear simulation model.

4.1 One Gate Controller

For the sake of controller design, the identification procedure discussed in 2.2.1 is used for building the model to be controlled, since the controller is designed for SISO systems. The procedure is performed with data from gate 3 and pool 3.

Since the controller is applied to a SISO system, the LQ performance index, used in the optimization, mixes the water level of the pool controlled and its respective gate, adopting the quadratic form as in equation (16). A scalar parameter weight, \( R \), can, then, balance the power needed for the input to control the water level. For the present situation, a Kalman observer is also designed to estimate the system state from the nonlinear process output. The observer gain is computed through the LTR principle [15], in which the covariance matrix of the measurement error is fixed and the covariance matrix of the gaussian disturbance is manipulated through a scalar parameter. By increasing this parameter, the loop gain and the stability properties of the LQG solution will approximate those of the LQ solution, assuming that there exists full access to the state.

Regarding the LQ control solution without observer and with full-state feedback, results for its simulation are presented in figure 23. Since the observer is not attached to the LQ controller, the linearized model is used to obtain such results. The parameter \( R \) that weights the input power for gate 3 is manipulated and different step responses are obtained. When the LQ weight is increased the closed loop system tends to slow down. Hence, the influence that the weight has on the control bandwidth is visible. The bandwidth is squeezed as the parameter is increased to spare the input energy.

However an undesirable effect does occur, that is, the overshoot value of the response slightly increases along with the increase of \( R \). If in continuous-time the converse effect is always observed, in discrete-time and particulary in this situation, this is possible since the controller is gradually approaching the location of the open loop poles. Figure 24 exemplifies the situation.
Figure 23: Simulation of the controlled linear SISO system that accounts gate 3 and pool 3, for different LQ weights. Full access to the state is used.

Figure 24(a) shows the closed loop zeros and poles for different LQ weights. Only the poles that are highlighted by the ellipse change their location, the others stay fixed in their open loop locations. For the sake of better observing the effect, this region is expanded in figure 24(b) and the same region for the open loop system is shown in figure 24(c). It can be seen that, as $R$ increases the poles move towards the open loop situation, a typical consequence of using LQ control. This has repercussions on the overshoot value of the system step response since the poles modulus increase to reach the open loop situation. Although the LQ optimization tries to remove the oscillation by decreasing the angle of the poles throughout the manipulation of $R$, the overshoot index is still compromised.

Figure 25 shows the changes, previously mentioned, in the loop gain bandwidth of the LQG controller for the manipulation of $R$. The reference accommodation integrator is included in this loop gain representation since it is integrated in the LQ performance index. Increasing $R$ results on the squeezing of the loop gain bandwidth and, consequently, the gain margin decreases, as can be seen in the magnitude diagram. Once again, the frequency plot for the system loop gain has a decreasing shape and does not include any resonant peaks due to the choice of lower order systems in the identification process, as discussed in section 2.2.1.

The SISO controller is then applied to the nonlinear MIMO system, in which only gate 3 is controlled and all others are fixed in their equilibrium positions, $34.2\text{mm}$, $39.6\text{mm}$ and $214.3\text{mm}$, for gates 1, 2 and 4, respectively. This must be in accordance with the overall equilibrium values for the MIMO system depicted in figure 26, with respect to the water levels along the canal. A step signal is applied to the reference for pool 3 with $50\text{mm}$ of amplitude.

Simulation results regarding the step response relatively to the desired SISO system reference are presented in figure 26, for three values of $R$. The test is performed on the nonlinear model. As expected, increasing $R$ slows down the response, a fact that is also verified for the simulation of the LQ controller with full-state access, discussed above.

The effects of unmodeled dynamics that are not accounted in the SISO system identification can be seen in this figure, as the step response presents oscillations in addition to the dominant behavior. Since the identification process is applied only to one pool at a time, the dynamics that come from the others are not taken into account and, so, they produce such effects on the step response. Nonlinear effects, that are not captured by the linear model can affect the response, as well. Due to this fact, the overshoot value is different for both, the nonlinear and linear cases. As $R$ is increased, the overshoot value in the nonlinear system’s step response decreases.

Although in this section the effects of $R$ are tested only for the centralized SISO controller, the conclusions presented can be extended to the MIMO centralized controller, both on the linear and nonlinear situation.

### 4.2 Four Gate Controller

In this section a four gate centralized controller is presented using the LQG approach. The conceptual scheme of the controller is presented in figure 27. It receives information from all pools and gates and manipulates every single gate at the same time. So, the canal is treated as a global MIMO system, that makes the LQG controller the global brain for the entire controlling process.
(a) Overall picture of the diagram. The area that is affected by the LQ weights is highlighted.

(b) Detailed picture of the highlighted area presented in figure 24(a).

(c) Location of the poles of the open loop system.

Figure 24: Zero/pole diagram in the Z-axis of the closed loop SISO system for different LQ weights.

Figure 25: Loop Gain frequency plot for different LQ weights.
Figure 26: Simulation of the controlled nonlinear SISO system, affecting gate 3 and monitoring pool 3, with different LQ weights. The set of equilibrium values used are 750mm, 600mm, 500mm and 300mm for the water levels in pools 1, 2, 3 and 4, respectively. This implies that pool 3 water level reference must be manipulated around 500mm.

Figure 27: Schematic of the LQG controller responsible for reading and controlling all water levels and gates.

The identification routine implemented to find the MIMO system for implementing such controller is discussed in 2.2.2. In this controller design, offtakes are not integrated in the solution, which means that only the $B$ matrix of the model (10) is identified and $\Psi$ is neglected.

In order to compute the LQ controller the same performance index structure applied to the SISO system controller is used (equation (16)) but, this time, the number of inputs (gates) and outputs (pool levels) is increased, as well as the weighting parameter $R$ that now has dimensions $[4 \times 4]$. This parameter is chosen with the goal of tuning the closed loop system step response relatively to the reference manipulation.

Likewise the previous section, a Kalman observer is designed, but in this case the LTR principle [15] cannot be applied. The LTR [15] condition relatively to the system being of minimal phase can not be ensured as there are no guarantees on where the system zeros are located. A study of the zeros can be done but is outside the scope of this work.

Thereby an adjustment of the LQG loop gain bandwidth must be made by manipulating the statistical properties of the signals noise, in such a way that it approximates the LQ loop gain bandwidth.

The statistical covariances of the disturbance and measurement noises are computed just as in the LTR [15] approach, but with the parameter $q$ responsible for the manipulation of the disturbance noise being adjusted. The covariances are defined as,

$$Q_0 = q^2 BB^T$$

$$R_0 = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}$$

(51)

where, $Q_0 [n \times n]$ is the covariance of the disturbance noise and $R_0 [4 \times 4]$ the covariance of the measurement noise.
(a) Singular values magnitude with the LQ controller and LQG controller for $q = 10^{-3}$.

(b) Singular values magnitude with the LQ controller and LQG controller for $q = 10^4$.

Figure 28: Loop gain magnitude over frequency of its singular values for a single LQ controller with full access to the state and two LQG controllers. Parameter $q$ manipulates the statistical properties of process disturbance noise.
Water level step responses for each of the four pools in the nonlinear system.

(a) Responses to the reference steps for each pool. LQ weights are set at: \( R_1 = 3500, R_2 = 2500, R_3 = 1500 \) and \( R_4 = 750 \), for the subsystems located at pools 1, 2, 3 and 4, respectively.

(b) Height of the gate 1 during the simulation.

(c) Height of the gate 2 during the simulation.

(d) Height of the gate 3 during the simulation.

(e) Height of the gate 4 during the simulation.

Figure 29: Simulation results for the nonlinear MIMO system with a LQG centralized controller. The set of equilibrium values used are 650mm, 500mm, 300mm and 150mm, for pool 1, 2, 3 and 4, respectively.
The choice of \( q \) is made through a trial process, such as to approximate the loop gain bandwidth to the LQ solution without observer.

Figure 28 shows a comparison for the loop gain bandwidth in both cases, the LQ controller with full-state access and the LQG controller (LQ with Kalman observer). The parameter \( q \) is fixed in \( q = 10^{-3} \) and \( q = 10^4 \) and the effects on the bandwidth are shown in figures 28(a) and 28(b). The best of both situations is represented in figure 28(b) where the LQG bandwidth almost approaches the LQ one. A bad example of a Kalman observer design is represented in figure 28(a) where the loop gain differs from the one obtained with LQ.

A simulation of the centralized LQG controller when applied to the nonlinear MIMO system is shown in figure 29. To carry out this simulation, the gate heights are set to hit the equilibrium situation for the water levels along the canal. Such levels are mentioned in figure 29 description. These heights are the basis for the functioning of the linear LQG controller and are fixed at: 34.34\( \text{mm} \), 34.82\( \text{mm} \), 38.41\( \text{mm} \) and 114.3\( \text{mm} \), for gates 1, 2, 3 and 4, respectively. With respect to the LQ weights used to design the controller, they are chosen in such a way that the cascade effects of the flow resulting from the opening or closing of gates are minimized. Given this, the assignment is done as described in figure 29(a), through a decreasing sequence from the first gate to the last. Thus, the first gate is the slowest of all four to react and the last gate the fastest. The last gate, can be as fast as possible since it has an overshoot drainage.

The simulation plot presented in figure 29(a) covers all the reference step responses from pool 1 to the pool 4 and the disturbance responses, that are triggered by each controller when water level slips its reference due to changes in the respective incoming flow.

In addiction, gate heights within this control time range are presented from figure 29(b) to figure 29(e). It can be seen that, if a water level changes in a pool that precedes a gate, the gate will have a stronger input response than if it changes in a pool that succeeds the gate. This is explained through the rate in which income and outcome flows vary relatively to the gate considered. The pool income flow of the gate considered changes directly with the gate movement but the outcome flow takes time to feel the effects of a successor gate movement. The outcome flow changes by the difference of pressures caused by the difference in water levels of adjacent pools.

### 4.3 Four Gate Controller fully decoupled

An uncoordinated version controller based on LQG is presented in this section for comparing its performance to the decentralized LQG controllers with optimal coordination, the purpose of this work. For this version to be absent of coordination among all gate/pool subsystems, each subsystem has a controller attached to itself that only sees its dynamics and its control actions, resulting in, four separated controllers. In the end of the design, four LQG SISO controllers are obtained, each one governing the movements of its associated gate.

The identification process is conducted in each subsystem, as if a single SISO system is to be identified, manipulating only the corresponding gate and fixing all others in an equilibrium situation. Details about the identification procedure are not discussed here since it is a matter of repeating the identification process for the SISO system provided in section 2.2.1, for each subsystem.

Controllers \( C_1, C_2, C_3 \) and \( C_4 \) and their interactions with gate/pool subsystems, pool 1, 2, 3 and 4 are schematized in figure 30. They are only allowed to read and give commands from its respective water level and to its respective gate actuator. As can be seen no kind of communication is established among them.

As for the identification process, the same controller design procedure and optimization index from the SISO controller case, discussed in section 4.1, are used. The LQ weights are again chosen through a decreasing scheme that is presented in the figure’s 31(a) description. As can be observed, this LQ weights are smaller than those used in the MIMO centralized controller (see previous section), where such weights could not be used. These weights would make the centralized MIMO controller much sensible to gate variations, with worse stability properties. The fully decoupled case can be design to be more faster than the centralized one because for each controller, other gates control signals are irrelevant when adjusting the height of its corresponding gate.

The Kalman observer can, as for the SISO system, be computed through the LTR principle [15], which simplifies loop gain bandwidth adjustment for the LQG.

Results on the simulation of the fully decentralized LQG controllers are shown in figure 31. Controllers are programmed with the gate equilibrium values needed to reach the water level stationary regime configuration presented on the description of figure 31. The set of gate values is assigned: 34.34\( \text{mm} \), 34.82\( \text{mm} \), 38.41\( \text{mm} \) and 114.3\( \text{mm} \). The same used in the LQG centralized controller case.
Figure 30: Schematic of the four LQG controllers with no coordination.

Figure 31(a) shows the step responses, as well as the disturbance responses due to movements in other gates. The simulation time and amplitude scale, used for the controller from section 4.2, are used. Figures 31(b) till 31(e) present the gates heights within this simulation time interval.

The major differences that are noticed from the centralized LQG approach to this one are focused on the disturbances rejection due to gate movements in other pools. As expected, the disturbance responses to movements in predecessor gates are worse since the absence of coordination implies the controllers to only react when slips in water levels are detected. These controllers do not take into account control signals from other gates than its own, which leads to a slower disturbance rejection than the centralized LQG solution. Figure 31(a) illustrates this fact. By focusing on the propagating disturbance responses through the canal, due to the reference transition in pool 1, it can be observed that water levels in pools 2, 3 and 4, firstly, began to fall and, only after that, they are compensated. Alongside, by analyzing the same situation in the figure 29(a), this time for the centralized LQG controller, the water levels of pools 2, 3 and 4 rise in the first place, meaning that gates 2, 3 and 4 reacted to the movement of gate 1. Also, by inspecting figures 29(c), 29(d) and 29(e), gate heights are less oscillatory than in figures 31(c), 31(d) and 31(e), between 130 and 145 minutes. Such a faster reaction compared to the previous improves disturbance rejections.

On the other hand, when disturbance rejection is due to movements in successor gates, the converse situation is experienced. The fully decoupled LQG controllers have better performance than the centralized LQG controller. Fully decoupled controllers alienate the fact that there is coupling between gates and, as the water levels do not slip much from the reference value, they do not compensate with the respective gate. It is shown in figures 31(c), 31(d) and 31(e) that the gate compensation are small compared with the centralized version (see figures 29(c), 29(d) and 29(e)). In the centralized version, the controller, by having full information of the canal and gates dynamics, has an excessive reaction.

With respect to the individual reference step response at each subsystem, it is expected that the centralized LQG controller behaves better since it has more information than the uncoordinated version. By analyzing both figures, 29(a) and 31(a), that is what actually occurs.

4.4 Centralized Four Gate Controller with feedforward control

A centralized MIMO system controller with feedforward control is presented here, which is useful as a template for comparing the decentralized feedforward control, in terms of performance. The case without feedforward control, that is discussed in section 4.2, does not serve as a term comparison because its performance is far below with respect to manipulation of offtakes. It is much limited due to the offtake action that, in section 4.2, is unmodeled and if the same LQ weights (R) are used in both controller designs, the solution without feedforward control will lead the nonlinear system to instability. As controllers presented in section 4.2 react only to water level changes and do not guess which effect will the disturbance have, they are a worse choice for the nonlinear MIMO plant, when manipulating offtakes. Slower controllers without feedforward control need to be designed, even for the plant not to be driven to instability.

Although the offtake disturbance problem was simulated in the controller without feedforward control to observe its effects, results on that are not shown since centralized controllers are not studied in detail in this work. The main objective of this section is to obtain a good template, comparing it with the decentralized case.

The system to be found for the current controller design is system (10). So, the identification process is similar to the centralized MIMO controller without feedforward control with the difference that the $\Psi$ matrix, responsible for the offtake modeling, is actually estimated. In fact, the controller referred in this section is in much alike to the
Figure 31: Simulation results for the nonlinear MIMO system with 4 LQG uncoordinated controllers, one for each gate/pool system. The set of equilibrium values used are 650mm, 500mm, 300mm and 150mm, for pool 1, 2, 3 and 4, respectively.
Responses to the disturbance by offtake 1 for each of the four pools in the nonlinear system

(a) Manipulation of offtake 1 and responses from each of the four pools.

Responses to the disturbance by offtake 2 for each of the four pools in the nonlinear system

(b) Manipulation of offtake 2 and responses from each of the four pools.

Responses to the disturbance by offtake 3 for each of the four pools in the nonlinear system

(c) Manipulation of offtake 3 and responses from each of the four pools.

Responses to the disturbance by offtake 4 for each of the four pools in the nonlinear system

(d) Manipulation of offtake 4 and responses from each of the four pools.

Figure 32: Results regarding the disturbance simulation for the centralized LQG controller with centralized feed-forward control. The nonlinear MIMO system is used and its equilibrium values are fixed at 650mm, 500mm, 300mm and 150mm, for pool 1, 2, 3 and 4, respectively. The intake flow is set at $Q = 20l/s$.

For designing such controller, the same LQ performance index used in 4.2 is optimized since gate positions are still the only manipulated inputs and water levels the process outputs as well. The LQ weights used are the same as in section 4.2, that is, $R_1 = 3500$, $R_2 = 2500$, $R_3 = 1500$ and $R_4 = 750$ for the gates 1, 2, 3 and 4, respectively. Although the optimal control law is computed through the same idea than the centralized LQG computations for the MIMO system, the co-state solution is changed in order to include the offtake action. A term is added to the co-state solution that does not depend on the process state but only on the disturbances signal. Given this, the term is referred as a feedforward term. The consequence of changing the co-state solution reflects itself on the overall control law, in which a term responsible for the feedforward arises, added to the state feedback control part. The final control law is, then,

$$u_{opt}(k) = -K_{ix}(k) - K_{fx}(k) + u_{off}(k)$$

where $u_{off}$ is the feedforward term that accounts the disturbances action from offtakes.

Details on the steps to integrate the offtake action in the optimal control law can be seen in appendix A. The decentralized feedforward part of the control law is identical to the centralized feedforward part, being that, in the present situation, the $\Psi$ matrix has a complete modeling of offtakes in the centralized case. Still, all the computational process remains unchanged.

The Kalman observer design procedure is the same as in section 4.2, where the statistical properties of the disturbance and measurement noise are manipulated so that the LQ loop gain properties are not spoiled.

Figure 32 shows simulation results obtained by varying the offtake inputs available in the nonlinear model.
same equilibrium conditions are chosen, as in section 4.2, such as to lead to the same set of gate positions.

In the simulation the reference is kept constant over all the time range and each offtake is manipulated at a different time interval with an input value of $10l/s$, to observe the disturbance response in all pools. This means that at each pool, where the offtake takes its action, $10l/s$ of water are removed from the overall process. The flow of water removed is important, as well as the place where it is removed. If the same flow of water is removed from the first pool and from the third pool, the last situation will represent a more demanding one.

The intake flow for the overall canal, in the present simulation, is fixed at $Q = 20l/s$, so these disturbances are considered to be demanding for the control problem at hand, as they are 50% of all the water flow that feeds the process. The responses in each pool are oscillatory, due to the gate oscillatory behavior, but in no way the stability is compromised. The response for offtake 3 (see figure 32(c)) is where the water level oscillation is more notorious. This is explained by the fact that this gate is the last undershot gate in the canal and, so, a disturbance at the end of the canal chain has more relevance on the dynamics than a disturbance next to the intake flow, as water takes its time to drain from the beginning to the end. When analyzing the disturbance responses for offtake 4 (see figure 32(d)), the situation faced is different since gate 4 is an overshot gate, which makes the drainage or the compensation process faster.

### 4.5 Pole Radius Constraint Action

In section 3.3, the main considerations in designing decentralized controllers for including the improvement of constraining the position of the poles were presented. Throughout this section results obtained by integrating such modification in the LQ optimization computations are shown for the centralized controller case.

Reminding the principle behind the closed loop poles constraint, an weighting function, $\alpha^{2k}$, is attached to the standard LQ regulator cost function. Thus, the weight, that varies over time, gives greater relevance to terminal states behavior than in the beginning of the control response, forcing the plant state to contract. Figure 33 shows the selected weighting function to produce such effects in the control regulation.

The parameter $\alpha$ is adjustable and is to be selected by fulfilling the condition $\alpha > 1$. Experiences with the closed loop poles constraint indicate that this parameter must be close to $\alpha = 1$.

Figure 34 illustrates the constraining process, where it is shown the location of a SISO closed loop poles in the zero pole diagram, when using a LQ controller with full-state access. These family of controllers are the same, as those discussed in section 4.1. The region shown is the part of the diagram that is affected by the optimal control optimization procedure. Two situations are overlapped in the figure, one with pole constraint and another without such improvement. The LQ weight $R$ is fixed in both situations.

As can be seen, when the pole constraint is integrated in the performance index the closed loop poles are placed inside a lower radius circumference. Thereby, as poles are shifted to the origin, the overshoot in the closed loop system response is reduced and rendered to be faster.

Furthermore, to observe the effects on a multivariate controller, the simulation results for the pole constraint
integration are shown in figure 35, when testing the centralized LQG four gate controller on the nonlinear model. For the test, all configurations in the controller design from figure 29, in section 4.2, are kept, as well as the equilibrium configuration for the canal. This test intends to show the step response performance on a single pool and how other gates respond to the disturbance, when there is no pole constraint and when it is applied. Thus, the reference for pool 1 is changed and pools 2, 3 and 4 take this reference transition as a disturbance. The pole constraint parameter is set to $\alpha = 1.02$. The water level regulation is shown on figure 35(a) and the gate control signals, that is, the actual gate positions may also be seen, from figure 35(b) till 35(e).

The remarks that can be made in the present comparison is that, regarding the individual step response, the controller is faster and the overshoot value slightly decreases, when using the pole constraint improvement. The controller reacts faster to changes in the reference and more abruptly, as can be seen through the manipulation of gate 1 in the figure 35(b). However, if the response of the controller placed in gate 1 yields good results on the individual step response, that is harmful to the disturbance compensation in other pools since abrupt movements in gates 2, 3 and 4 lead to a worse performance. Although the disturbance response at each pool is done in a short time period, the water level deviation from the desired reference is slightly higher.

The LQ controllers, for each situation of $\alpha$, that are used in the simulation results from figure 35, are, hereafter, analyzed in the frequency domain. Figure 36 shows the singular values plot of closed loop systems over frequency in which these controllers are integrated. This provides an enlightening analysis on the effect of the pole constraint in MIMO systems, with respect to changes in the bandwidth and in the overshoot level.

The closed loop system is computed including the LQ controller with full access to the process state and, thereby, the Kalman observer is not considered. On the left hand side (figure 36(a)), the overall frequency analysis of the singular values is shown, for the pole constraint case and for the simple centralized LQ controller. The static gain of each singular value is $0 dB$ due to the transfer function being considered to be the closed loop system transfer function. From the left hand side figure, the presence of resonant peaks at higher frequencies is not clear, so, a magnified view on the region of interest is presented in figure 36(b). In this figure, the expected system behavior in applying the pole constraint modification is emphasized. The bandwidth is extended when the pole constraint parameter is increased and, also, the resonant peak decreases, thereby making the signal response of the closed loop system faster and with less overshoot.

4.6 Model Order Reduction Vs Compensator Order Reduction

Sometimes complex linear systems, that imply the usage of complex linear controllers, are hard to deal with, since they could be much demanding in terms of computational costs and harder to implement. Or, when integrating the software, complex systems can be more sensible to induce bugs or errors, that cause problems to software maintenance. Therefore, most of the times, simpler systems that do not compromise the performance are preferred than
Figure 35: Simulation results with respect to the responses in the presence of a reference step for pool 1. Results with pole constraint ($\alpha = 1.02$) and without pole constraint ($\alpha = 1$) are overlapped in the same figure. The centralized LQG four gate controller is used.
(a) Overall representation of the singular values plot.

(b) Amplified representation of the singular values in the resonant peaks zone.

Figure 36: Singular values of the closed loop MIMO system over frequency, for pole constraint ($\alpha = 1.02$) and without pole constraint $\alpha = 1$. The LQ controller is used with full-state feedback.
others that have a proficient modeling. This leaves the decision to the designer towards the trade-off, simplification versus performance, that is, whenever the performance of lower order systems do not go below an acceptable standard.

What is intended with this section, is to show two distinct options in simplifying the current control methodology to a highly compacted form and show that these simplifications can be applied to the given problem. Although the controller considered in this section is only the centralized MIMO controller, these reduction methods can be implemented in decentralized controllers presented in section 3.1.

The LQ weights chosen for both controller designs, in the model order reduction and in the compensator order reduction, are \( R_1 = 35000, R_2 = 25000, R_3 = 15000 \) and \( R_4 = 7500 \), for regulation of gates 1, 2, 3 and 4, respectively. The Kalman observer is also adjusted to provide the closest approximation possible to the LQ loop gain bandwidths.

Furthermore, a model order reduction and a compensator order reduction are done through a Gramian-based balanced realization of these systems. A compensator reduction implies that the original model identified from a target process is maintained and only the compensator order is changed with direct implications in the controller behavior. On the other hand, in a model reduction, the model to be reduced is the one obtained through identification of the target process and, so, the controller design is, consequently, affected by the reduced model’s view of the process. Even, by assuming that the same orders are chosen for both cases, the final compensators that will regulate the nonlinear process are similar with respect to their orders, they produce different results. Due to the limitations that a simplification of the linear model can bring to the controller design, the compensator order reduction is always preferred [17]. Further ahead both situations will be compared.

To implement an order reduction, in both cases, an analysis of the Hankel singular values [17] is performed to each state-space system depending on the situation, linear model or compensator’s system. The Hankel singular values measure the contribution that each state has on the input/output behavior of the state-space system considered. This analysis is useful for choosing a better approximation of the linear system by inspecting the states contribution and whenever a state can be discarded by having lower contributions to the system’s output.

Regarding linear model order reduction, figure 37 shows the Hankel singular values of the centralized MIMO system identified for the centralized MIMO controller design in section 4.2, in terms of energy contribution of each state. A threshold is placed in the bar plot, represented by a red line, that bounds the states to be discarded. A demanding reduction is presented since the system is truncated from 16 states to 4 states, for describing the overall canal dynamics.

Now, with respect to the compensator order reduction the same procedure is adopted, but this time the system used is the compensator, which means that the linear MIMO model remains unaffected. The Hankel singular values can be seen in the figure 38, as well as the compensator’s threshold line, again bounding the states to be discarded. After having an idea on the approximation order for both systems, a balanced realization is found for each system, through their observability and controllability gramians. The realization method can be consulted in [18]. Therefore, the less informative states can be removed, considering their hankel singular values energy to input/output attachment. In the case of the model order reduction, a controller design is, hence, made upon the new model with the same performance characteristics than in the design for the compensator order reduction.

Therefore, to see how good each approximation is, the loop gains with final compensators are computed for each case. The standard centralized MIMO controller’s loop gain is, also, computed to have a comparison degree. Once more, the same LQG design parameters and criterions are used for the comparison to makes sense. The analysis of the compensators loop gains through the plot of their singular values over frequency is presented in figure 39.

A first remark to the loop gain analysis is that, for lower frequencies all three overlap each other, except the mode with lower bandwidth from the linear model order reduction, meaning that lower frequencies are unaffected by the reduction. This is in concordance with the fact that higher order systems are used to describe higher frequency dynamics. Furthermore, higher frequencies are better approximated by the reduced compensator option, as the bandwidth of its singular values approach those of the standard centralized LQG controller. A similar reduction done in the linear model leads to a bigger loss in the loop gain bandwidth. This confirms the fact that a compensator order reduction is preferred.

Another way to inspect the performance of both solutions is to observe their responses to a reference step transition. Figure 40 shows the closed loop responses of each pool by changing their references at a time.

In the reduced model, gate 1 is unable to stabilize the water level, as well as for other gates, when they succeed to stabilize their water levels, this is done within a large time range and a higher overshoot value by comparing it
Figure 37: Hankel singular values plot of the state-space MIMO system with the state energy threshold overlapped.

Figure 38: Hankel singular values plot of the MIMO LQG compensator with the state energy threshold overlapped.
Figure 39: Singular values plot of the loop gain in all three situations: when the MIMO process system and LQG controller are not reduced; when only the MIMO process’s system is reduced; and when only the LQG controller is reduced. Overall compensator (LQ controller plus Kalman observer) is considered in the loop gain plot.

Figure 40: Responses to step reference in the pool 1 at each of all four pools for all three controller design situations.
with all other two situations. What causes this to happen is the low knowledge that the lower order models have on the process and, so, gates fail to coordinate or to actuate at the due time.

Regarding the reduced LQG controller, it behaves well for pool 2, 3 and 4, but in the pool 1 the performance degrades, relatively to the centralized LQG controller. Also, when changing the reference level in a given pool, the responses in their successor pools are much oscillatory than the centralized LQG controller. However, as pointed out previously, this is a demanding example in which the compensator only works with 4 states, while the linear model identified has 16 states.

5 Decentralized Control Design

Through this chapter, results on the decentralized LQG controller with coordination are discussed and the nonlinear model is used as the target model for control purposes. This nonlinear model was also used for centralized controllers, presented in the previous chapter. Thereby, performance comparisons are made between the centralized and decentralized solutions, both in terms of reference manipulation and offtake manipulation. Furthermore, the decentralized controller response is analyzed for different LQ weights and in what way the response changes when the closed loop pole constraint improvement is integrated in the controller design.

The family of controllers that are addressed in this chapter are only designed for the 4 gates/4 pools case, meaning that all gates and pools are available for control. Although controller versions for systems with less number of inputs and outputs are discussed in the previous chapter and tested on the experimental canal, in the present chapter these versions are not covered.

Within the design of decentralized LQG controllers the identification procedure discussed in section 2.2.3 is performed, in order to find the linear model with which controllers are to be designed. To remember, this system embeds the coupling dynamics between successive gates along the canal, so that each pool level depends on the gates position located at neighboring pools. Offtakes are integrated in the decentralized control law through the same principle as for gate actuation. The linear system considers that offtakes located inside the neighborhood of a given offtake influence the pool level where the offtake is located. With respect to the canal standalone dynamics, these are represented through a complete decoupled form and each pool level is computed accounting only its level information, and neglecting the fact that water levels along the canal are dynamically interconnected. Under these assumptions, a decentralized structure is obtained, for controller design.

The overall decentralized controller design is presented in section 3.1 together with appendix A, in which the steps to reach the solution are presented in more detail. For the sake of remembering what is involved in the decentralized control, it is observed that the following full optimal control law is used for the decentralized case,

$$u_{opt}(k) = -K_{fb}x(k) - K_Ix_I(k) + u_{gate}(k) + u_{off}(k)$$  \(53\)

where \(K_{fb}\) and \(K_I\) are the state feedback gain of the process and the state feedback gain of the integral effect, respectively. These gains are computed in such a way that each gate position only accesses its state dynamics. The signal \(u_{gate}\) represents the coordination signal to be applied to the gates, that is the main component in the decentralized control law, responsible for achieving the coupling based solution. The signal \(u_{off}\) provides the feedforward control signal computed in a decentralized fashion, since offtakes are not explicitly connected in terms of information exchange. Decentralized feedforward control provide an increased performance towards offtake manipulation that controllers without feedforward do not have.

Regarding the observer that is needed for controlling the nonlinear process, a Kalman-Bucy estimator is designed as described in section 3.5. In the actual case, as the state feedback solution is complete decoupled, the Kalman observer gains are computed separately for each local controller. This means that the LTR(Loop Transfer Recovery) principle [15] is used as an approximation. The statistical properties of the noise are, thereby, defined in accordance with LTR.

Although several frequency analysis are performed in the previous chapter on the centralized controller case, in the decentralized controllers case this is not so straightforward due to the extra term that the coordination method adds to the overall solution.

At each sampling time instant, the coordination method is run in an iterative process. Thereby the coordination
By observing figures 43(a) till 43(b), it can be verified that the response performance slightly improves in the step figure 43(a) till figure 43(d). The simulation is run on the non linear model. Results on the simulation, where references for all pools along the canal are manipulated are shown in figure 41. The set of gate equilibrium positions that are used to simulate the nonlinear model are: 34.34mm, 34.82mm, 38.41mm and 114.3mm, for gates 1, 2, 3 and 4, respectively. These gate positions, in turn, lead to the water level configuration provided in the description of figure 41.

With respect to the LQ weights assigned during the controller design, in the decentralized case they must be bigger than for the controllers presented in the centralized control chapter. This is due to the fact that there is a consensus among controllers and it accelerates the output regulation, even in the absence of a centralized control law. If the LQ weights are as smaller as centralized situations, this will lead the nonlinear model to instability. Therefore, higher LQ weights are chosen and they are assigned in a cascade fashion with controller for gate 1 being the one to have a higher value, as in other centralized designs.

The weight configuration depicted in figure 41(a) description produces the best results relatively to the individual step reference responses and disturbances rejection caused by reference manipulations. Figure 41(a) shows water levels throughout the canal when references at each pool are changed by a step signal. Figures 41(b) till 41(e) present the gates position in the same time interval as shown in figure 41(a) to observe their behavior. A small oscillation in the position of gates 3 and 4 (see figures 41(d) and 41(e)) is observed, caused by the fact that the controller bandwidth is still large and the LQ weights should be increased. Nevertheless, water levels follow the reference with no oscillation.

To perceive the influence that LQ weights have on the coordinated controllers performance, simulations are performed for three different LQ weights that differ from each other by a multiplicative factor. They are chosen based upon the cascade set of weights: $R_1 = 700$, $R_2 = 500$, $R_3 = 300$ and $R_4 = 150$, for controller 1, 2, 3 and 4, respectively, that is referred as $R_{base}$, in which pool 1 has the biggest weight value. Therefore, the LQ weights, used in simulations, are chosen multiplying $R_{base}$ by 100, 500 and 1000. As the multiplier is increased the assignment has a dispersive effect on $R_{base}$ rendering controllers located at the head of the canal to be even more slower than those located at the end.

Furthermore, in the simulation only the reference for pool 2 is changed and the corresponding step response is observed, as well as other responses over the canal due to this manipulation. Results on the simulation referred are shown in figure 42. Figures 42(a) till 42(d) present the individual picture of each pool level over the same time range of pool 2 step response. As expected, when the multiplicative factor is increased the step response rate in pool 2 decreases considerably and the overshoot value is smoothed. Although there is no frequency analysis tool to evidence this fact, it can be seen that the controller bandwidth is squeezed by observing the closed loop response, when their LQ weights are increased.

Also, by observing the disturbance rejection performed by controllers 1, 3 and 4, it can be seen that the compensation is done in a smaller time interval when weights are decreased. Relatively to the reference slip, a stronger controller coordination (when weights are decreased) lead to a smaller level slip at the end of the compensation but in the beginning it is experienced a more abrupt transition than for slower controllers.

Furthermore, tests on the constraint of closed loop poles placement are performed to check in what way the performance of coordinated controllers in controlling the nonlinear model is affected. To make the comparison between the cases when there is pole constraint and when there is not, the decentralized controllers are designed in both situations with the same set of LQ weights: $R_1 = 40000$, $R_2 = 30000$, $R_3 = 20000$ and $R_4 = 5000$, for controllers 1, 2, 3 and 4, respectively. To the pole constraint parameter, $\alpha$, are assigned the values $\alpha = 1.004$ and $\alpha = 1$, that represent the situation where exists pole constraint in the controller design and the situation where such improvement is absent.

The reference for pool 2 is changed at the same simulation time instant for comparing the step response when including both controller designs. Results on the step reference response in pool 2, as well as the resulting disturbance compensation are shown in figure 43. Water levels behavior throughout the canal can be observed from figure 43(a) till figure 43(d). The simulation is run on the nonlinear model.

By observing figures 43(a) till 43(b), it can be verified that the response performance slightly improves in the step
Water level step responses for each pool in the closed loop nonlinear model

(a) Responses to the reference steps for each pool. LQ weights are set at:
$R_1 = 7000$, $R_2 = 5000$, $R_3 = 3000$ and $R_4 = 1500$, for controllers 1, 2, 3 and 4, respectively.

(b) Position of gate 1 during the simulation.

(c) Position of gate 2 during the simulation.

(d) Position of gate 3 during the simulation.

(e) Position of gate 4 during the simulation.

Figure 41: Simulation results for the nonlinear MIMO system with the decentralized LQG controller. The set of equilibrium values used are 650mm, 500mm, 350mm and 200mm, for pool 1, 2, 3 and 4, respectively.
reference change for pool 2 and in the compensations at pools 1, 3 and 4, when pole constraint is used. The step response in pool 2 becomes faster with no overshoot at all and the outlooks in pools 1, 3 and 4 are improved. These water level compensations are performed within a smaller time interval and reference slips are minimized. With the controller design configuration mentioned, this is the highest value for \( \alpha \) that can be assigned. In theory, if the parameter is increased, the responses will be improved since closed loop poles are constrained to a circumference shifted towards the origin. However beyond \( \alpha = 1.004 \) the nonlinear model will be driven to an oscillatory state or even to instability.

5.1.1 Coordination between agents

In order to evidence the distributed coordination algorithm throughout all controllers a simulation on the nonlinear model is run in which the reference for pool 3 is changed. At that very time instant, that controllers start to react to the reference step, a picture is taken to the manipulated variables, that is, the controllers output signals. The consensus between controllers can be seen in figure 44 within the sampling time interval referred. Two cases are shown in this figure. The situation where low LQ weights are used in the controller design (see figure 44(a)) and other with higher LQ weights (see figure 44(b)) for comparing the coordination scheme in both cases. Between simulations only the weights are changed, being that both controller designs are much alike differing only on the weights assignment. The weights set for both cases are presented in figures 44(a) and 44(b), for the low weights case and for the high weights case, respectively. When low LQ weights are used the coordination is more oscillatory and takes more iterations to converge, a similar behavior to the effect that LQ weights have on step reference responses (see figure 42). A remark must be done as gates 3 and 4 are the most oscillatory of all four. This is explained through the fact that the reference is changed in pool 3, therefore, consecutive gates have strongest gate signals since the flow difference produced by the response affects directly the succeeding pools. Thereby controllers located at pool 3 and pool 4 need to plan better their strategies to tackle the gates actuation problem.
(a) Response to the step reference from pool 2 in the pool 1 for two different controllers designed with pole constraint parameters: $\alpha = 1.004$ and $\alpha = 1$.

(b) Step reference response in pool 2 for two different controllers designed with pole constraint parameters: $\alpha = 1.004$ and $\alpha = 1$.

(c) Response to the step reference from pool 2 in the pool 3 for two different controllers designed with pole constraint parameters: $\alpha = 1.004$ and $\alpha = 1$.

(d) Response to the step reference from pool 2 in the pool 4 for two different controllers designed with pole constraint parameters: $\alpha = 1.004$ and $\alpha = 1$.

Figure 43: Simulation results for the nonlinear MIMO system with the decentralized LQG controller with respect to the reference change in pool 2. A comparison for the usage of pole constraint ($\alpha = 1.004$) and nonexistence of pole constraint ($\alpha = 1$). The LQ weights $R_1 = 40000$, $R_2 = 30000$, $R_3 = 20000$ and $R_4 = 5000$, for controllers 1, 2, 3 and 4, respectively, are used for both.

Figure 44: Agent coordination between two sampling time instants immediately after the reference for pool 3 was changed. The controllers for each case are designed with different LQ weights.

(a) LQ weights configuration: $R_1 = 70000$, $R_2 = 50000$, $R_3 = 30000$ and $R_4 = 15000$, for controllers 1, 2, 3 and 4, respectively.

(b) LQ weights configuration: $R_1 = 7000000$, $R_2 = 5000000$, $R_3 = 3000000$ and $R_4 = 1500000$, for controllers 1, 2, 3 and 4, respectively.
5.2 Integrating Accessible Offtake Action

By using the control law described as well as all the considerations made in the beginning of the chapter, the off-take action integrated in the coordinated controller design as a feedforward component. Also, as mentioned above, the systems for which the controllers are designed do not have an overall modeling of the dynamics introduced by offtakes, that is, the dynamics modeled are by itself a constrained version of the centralized offtake version. Therefore, the performance of the decentralized case will be always rated below the centralized case.

Results on the simulation performed with the nonlinear model are shown in figure 45. In the controller design the assignment of LQ weights is done through the following order: \( R_1 = 70000, R_2 = 30000, R_3 = 50000 \) and \( R_4 = 1500 \), for controllers 1, 2, 3 and 4, respectively, and the improvement through pole constraint is disabled for the decentralized and centralized cases (section 4.4) may be comparable. In the simulation the set of values corresponding to the equilibrium positions for gates, that lead to the water level equilibrium configuration depicted in figure 45 description are: 34.34mm, 34.82mm, 38.41mm and 114.3mm, for gates 1, 2, 3 and 4, respectively and the intake flow of the process is set at 20l/s. Throughout all simulation time the references are kept constant and only offtakes are opened and closed, one at a time, so that offtake disturbances do not overlap. Offtakes are opened with 10l/s of ejection flow.

Furthermore for the water levels to converge, the feedforward control component is multiplied by a scalar, \( \xi \), that regulates the amount of feedforward control that is added to the overall control law. Thereby this scalar needs to be in the range \( 0 < \xi < 1 \). Without this, pool levels would be driven to instability. In the present simulation the assignment \( \xi = 0.6 \) is done, that is, only 60% of the feedforward control is introduced in the coordination procedure. This is a way to reduce the relevance of feedforward control in the gates manipulation, which could be avoided if a more complete modeling of offtakes was provided. However, in the present decentralized controller structure such modeling is not possible and any decoupled model version for offtakes will always represent an approximation.

Figures 45(a) till 45(d) show responses throughout the canal to disturbances induced by offtakes 1, 2, 3 and 4, respectively. The major remark to be done is that pools located prior to the pool where the offtake is manipulated, have an almost cleaned disturbance rejection. This is explained through the decentralized architecture of these coordinated controllers, in which only neighboring controllers exchange information with each other, leaving others that are outside the neighborhood without the need of adapting their strategy. Same effects can also be identified in the step reference responses presented in figure 41.

5.3 Performance Analysis

In this section performances for the centralized LQG controller, the fully decoupled controllers and the decentralized LQG controllers are compared regarding the reference step response at each pool. Later on, the disturbance rejection introduced through offtake action is also discussed when using the centralized feedforward control and the decentralized feedforward control.

Regarding the reference step response, the centralized LQG controller, the fully decoupled version and the decentralized LQG controllers are presented in figures 29, 31 and 41, respectively. Analyzing their performances, it is remarked that the decentralized control case takes advantage in both other control approaches for different scenarios.

Focusing on the individual reference step response in each controller case, the decentralized control produces better results than the uncoordinated case since the overshoot is reduced and the same response rate is kept. But results are inferior, although very close, when comparing with the centralized LQG controller. Concerning the gates manipulation at the time range when controllers react to their references change, the case where are experienced strongest gate manipulations is the uncoordinated case, as expected, since these kind of controllers in face of slips between the reference level and the actual pool level. Thereafter, decentralized controllers, are located a step below relatively to the uncoordinated case as they have weaker gate manipulations, which makes the centralized controller, the one to have the smoothest gate manipulation.

With respect to level compensations in predecessor pools when facing a reference step response, controllers from the uncoordinated version are those showing the best results with almost no reference slips due to the fact that these controllers neglect their gates manipulation and leave pools to stabilize its level almost autonomously. This time, the centralized version is the one to have the poorest performance of all, for the present situation. Due to the information exchange about gates positions across all controllers, the commands that they sent to their gates
Responses to the disturbance by offtake 1 for each one of the four pools.

(a) Manipulation of offtake 1 and responses from each one of the four pools.

Responses to the disturbance by offtake 2 for each one of the four pools.

(b) Manipulation of offtake 2 and responses from each one of the four pools.

Responses to the disturbance by offtake 3 for each one of the four pools.

(c) Manipulation of offtake 3 and responses from each one of the four pools.

Responses to the disturbance by offtake 4 for each one of the four pools.

(d) Manipulation of offtake 4 and responses from each one of the four pools.

Figure 45: Results regarding the disturbance simulation for the decentralized LQG controller with decentralized feedforward control. The nonlinear MIMO system is used and its equilibrium values are fixed at 650mm, 500mm, 350mm and 200mm, for pool 1, 2, 3 and 4, respectively.

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<tr>
<th>Weights</th>
<th>Centralized version</th>
<th>Decentralized version</th>
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<tr>
<td>$R_1$</td>
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<td>$R_2$</td>
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<td>$R_4$</td>
<td>750</td>
<td>1500</td>
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Table 1: LQ weights used for both, centralized and decentralized versions.

are exaggerated and, therefore, bigger reference slips are noted. This behavior can be checked by observing the gates manipulation in figure 29, at the time intervals where the referred compensations occur.

In turn, the performance of the decentralized version is placed between these two previous discussed versions, being that, it behaves better than the centralized LQG controller but not as better as uncoordinated controllers.

By analyzing level compensations in successive pools when a reference is changed in a pool located at the beginning of the canal chain, the version that has the best performance, as expected, is the centralized LQG controller. A full coupling scheme produces better results than a partial or an nonexistent coupling and, so, the uncoordinated version are the ones that more degrade the compensation performance.

However, by comparing decentralized LQG controllers with the centralized LQG controller is noted that, the performance of decentralized controllers are not far from the centralized version.

A comparison is also performed between the centralized and the decentralized feedforward control performances, both illustrated in figures 32 and 45, respectively. By observing the results it is noted that the decentralized version has a better disturbance rejection. The compensation is done in a shorter time interval and with less reference slips, even with bigger LQ weights. Table 5.3 shows the set of LQ weights used in both versions.

The fact that the decentralized controllers are able to reject the disturbances in a short time interval with bigger weights is explained through the coordinative scheme that accelerates the level compensation process and, so, they are preferred when dealing with offtakes than a full centralized representation of the canal. This last option does not have an adaptive behavior.
6 Experimental Canal Testing

This chapter presents some experiments performed in the physical canal located in the NuHCC facilities at Évora University. As done in centralized and decentralized control chapters, here are presented results for SISO controllers applied to single gate/single pool systems, MIMO controllers applied to 4 gates/4 pools systems and several design situations for the decentralized controllers when applied to 4 gates/4 pools systems. The decentralized controllers are then tested for several controller design cases: by manipulating LQ weights (R), by manipulating the closed loop poles constraint parameter (α) and by regulating the offtake feedforward parameter (ξ).

Furthermore, an experience with controller order reduction is presented with 3 gates/3 pools centralized controllers.

For all controllers previously mentioned two situations are tested, such as reference manipulation and offtake manipulation.

The experimental tests performed at NuHCC, that are presented above, would be impossible to obtain if it had not been carried out a software design for the MATLAB/SCADA interface [2]. With this interface gates, offtakes and MONOVAR manipulation is made possible, as well as receiving readings on the current pools water levels. This information exchange both, in receiving and retrieving data, is done within a relatively short latency times, at least less than the sampling time interval used by controllers.

Although work done on the nonlinear simulator [8] only attends SISO and MIMO controllers, that is, 1 gate/1 pool and 4 gates/4 pools controllers (see chapter 4), centralized controllers for 2 gates/2 pools and 3 gates/3 pools systems are tested on the experimental canal but their results are not shown in the current chapter. They can be consulted in the appendix E.

Also, feedforward control was only tested for decentralized architectures.

For the design of controllers used in experimental tests, same identification procedures used in the centralized and decentralized chapters are used, performed with the nonlinear model data and not with experimental data. Throughout the field tests performed, it was noticed that the nonlinear simulator data is enough to describe the experimental canal and test these controllers but if a high performance controller is desired, experimental data should be retrieved for identifying linear models that achieve such performances.

In the controllers design it is included a command saturator for the gates since they have input limits beyond which the commands are considered invalid. So, for the controller not to ramble above or below 800mm and 0mm, respectively, a saturator that unloads the error integrator is used.

Furthermore, for improving the reference tracking, a one-pole filter is applied to the reference. This smooths the solution instead of having an instantaneous transition which helps controllers to gradually achieve an equilibrium.

A especial remark must be done to the last gate since its input behavior mismatch both, on the nonlinear simulator and on the experimental canal. On the nonlinear model, openings in the gate (by decreasing its height) will drop the pool 4 water level meaning that the gate position is measured from the bottom of the canal. Furthermore, by inspecting the experimental readings obtained on the experimental canal, it is concluded that the gate position is measured from the upper bound of the actuator since gate 4 opening commands (by decreasing its height) lead to an increase on the water level. This mismatch is solved by affecting a minus signal to the commands sent to gate 4 in the identification process when using the nonlinear model data on the experimental canal controller design. Since the identification process works with incremental data, that is, offset values are neglected, both nonlinear and experimental canal input scenarios for gate 4 are symmetric.

6.1 Centralized Control

This section presents results on tests where centralized controllers are used. Results on two designs for 1 gate/1 pool LQG controllers with different LQ weights are shown, results on a 4 gate/4 pool centralized LQG controller are provided and, finally, the compensator order reduction, already discussed in section 4.6, is tested on a 3 gate/3 pool centralized LQG controller.

Experimental simulations on 2 gate/2 pool and more 3 gate/3 pool centralized LQG controllers were also performed during the field tests and can be seen in sections E.1 and E.2, respectively.
6.1.1 One Gate Controller

Figure 46 and 47 show results for LQG controllers applied to the SISO system accounting gate 2 and pool 2. For both cases all gates are kept constant except gate 2 that is manipulated and off-takes are not manipulated during the simulation. A reference step is applied to the closed loop system to see its step response.

In the case depicted in figure 46 the lowest LQ weight controller design is presented, from the two LQ designs shown here. The water level configuration presented in the description of the given figure, obtained by setting the gates position at 100mm for gate 1, 2 and 3 and 350mm for gate 4. Intake flow contributes with 25l/s of income flow to the overall process.

A LQ weight parameter $R = 200\sqrt{1000}$ is assigned to the controller performance index. Figure 46(a) shows the water level response for two steps, one ascending and another descending, applied to the pool 2 reference. The set points correspond to a 10mm increment on the previous water level and a 20mm decrement for the second step. Figures 46(b) and 46(c) present the gate manipulation by the SISO controller during the simulation and the error measured between the current water level and the imposed reference, respectively.

Figure 47 shows the higher LQ weight controller design case, again, for the SISO system composed by pool 2 and gate 2. The equilibrium regime for all water level throughout the canal is set as mentioned in the description of figure, that corresponds to the following gates positions for gate 1, 2, 3 and 4: 100mm, 100mm, 100mm and 400mm. Intake flow is set at 25l/s.

The LQ weight assignment that leads to these simulation results is $R = 500\sqrt{1000}$. The reference step response can be seen in figure 47(a), in which a set point change of +20mm is applied to the pool level. Figures 47(b) and 47(c) show the gate position and the error between the reference and pool level read, over all time range.

As can be seen in both figures, if the LQ weight is decreased the step reference response becomes faster but more oscillatory (see figures 46(a) and 47(a)). It can be seen that a lower LQ weight design reaches a zero error state in less time, by observing figures 46(b) and 47(b). Nevertheless by observing the actuators signals from both cases, the increase of the response rate can be verified in figures 46(c) and 47(c), so gate 1 is more oscillatory than the case of the higher LQ weight design.
(a) Step reference time response of pool 2 water level. The LQ weight is set at $R = 200\sqrt{1000}$.

(b) Position of gate 2 during the simulation.

(c) Error signal between the level in pool 2 and the reference during the simulation.

Figure 46: Results on the experimental canal regarding the reference manipulation of pool 2 with the LQG controller for SISO systems. The equilibrium configuration throughout the canal is: 645mm, 585mm, 542mm and 428mm, for pool 1, 2, 3 and 4, respectively.
(a) Step reference time response of pool 2 water level. The LQ weight is set at $R = 500\sqrt{1000}$.

(b) Position of gate 2 during the simulation.

(c) Error signal between the level in pool 2 and the reference during the simulation.

Figure 47: Results on the experimental canal regarding the reference manipulation of pool 2 with the LQG controller for SISO systems. The equilibrium configuration throughout the canal is: 607mm, 541mm, 467mm and 320mm, for pool 1, 2, 3 and 4, respectively.
6.1.2 Four Gate Controller

Experimental results when using a centralized LQG 4 gate/4 pool controller are shown in figure 48. In this simulation the initial configuration for the gates position chosen is 100mm for gate 1, 2 and 3, 399.7mm for gate 4. When left to stabilize, the equilibrium configuration used is, therefore, translated in the water level values at pools 1, 2, 3 and 4 that are presented in the description of figure 48. The intake flow used is $Q = 24l/s$.

For the controller design, the LQ weights depicted in the description of figure 48(a) are used as well as closed loop poles constraint by choosing $\alpha = 1.009$.

In this test the set point for each pool is risen to an upper level of +50mm, beginning from pool 1 to pool 4, and the level at each pool is left to stabilize before making any set point changes in other pools. By observing figure 48(a) it can be seen that the time responses at each pool are slightly oscillatory which is caused by the type of controller design used and errors when modeling the process dynamics that cannot be avoided. Such oscillations can also be verified by the manipulation of gates position (see figures from 48(b) to 48(e)), where the controller tries to stabilize each pool with much oscillatory control inputs sent to gates.
(a) Time responses to the reference steps for each pool. LQ weights are set at: $R_1 = 4000$, $R_2 = 2500$, $R_3 = 2000$ and $R_4 = 100$, for actuation of gate 1, 2, 3 and 4, respectively. Closed loop poles constraint is used with $\alpha = 1.009$.

(b) Position of gate 1 during the simulation.

(c) Position of gate 2 during the simulation.

(d) Position of gate 3 during the simulation.

(e) Position of gate 4 during the simulation.

Figure 48: Results on the experimental canal regarding the centralized LQG controller for 4 gates/4 pools systems. The equilibrium configuration throughout the canal is: 537 mm, 495 mm, 473 mm and 386 mm, for pool 1, 2, 3 and 4, respectively.
6.1.3 Compensator Order Reduction

In this section experimental results are shown regarding a compensator order reduction when applied to a given LQG compensator. Thereafter the original compensator and its reduced version cases are presented for comparison purposes. The controllers used are centralized and only embrace 3 gate/3 pool systems, being that the last gate is outside the controller range, so is kept constant.

In figure 49 the original compensator results are observed using 9 states for regulating the three controlled pool levels. These states are all the states that are available from the identified linear model. The initial gate position configuration is: 100\text{mm} for gates 1, 2 and 3, and 400\text{mm} for gate 4, which corresponds to the water level configuration for the experimental canal depicted in the description of figure 49. The intake flow that feeds the process is kept constant at $Q = 26.5\text{l/s}$.

Regarding the controller design, LQ weights are chosen as described in the description of figure 49(a) and the closed loop poles constraint is not used.

For obtaining the results on figure 50, the previous compensator is then reduced through the same way as done in the simulator results obtained for centralized LQG controllers (see section 4.6). The reduction is done by decreasing the number of work states used by the compensator from 9 to 3.

By previous compensator it is understood that same specifications are used from the original controller for this controller design, such as LQ weights and pole constraint parameters, in order to separate the compensator reduction effects from those of a different design.

Regarding the initial gates position, the following configuration is used: 100\text{mm} for gates 1, 2 and 3, and 400\text{mm} for gate 4. This leads to the water level values indicated in the description of figure 50. The intake flow is again set at $Q = 26.5\text{l/s}$.

In both situations, as seen in figures 49(a) and 50(a), the set point of pool 1 is risen with a 50\text{mm} increment and the same is done to next pools, 2 and 3. After all three pools are stabilized on the upper level, the set point for pool 1 is decreased by 100\text{mm}.

As the set point value is decreased a curious situation is observed in figure 49(a). The set point for pool 2 exceeds pool 1 set point, meaning that it is an impossible configuration since the normal water course is when pool 1 drains towards pool 2 and not in reverse. This causes the pool 1 to never catch its reference set point and as result, gate 1 will be opened infinitely until reaching its limit. Figure 49(b) shows the noted situation, where gate 1 begins to open at 160 minutes and continues to open till the simulation is terminated.

This fact leads to the conclusion that the saturation limit for the gates position cannot be the maximum opening values but the values when gates are no longer submerged. And that must be considered for the sake of a fast recovering controller when facing such incorrect set point configurations.

By comparing the behavior of both solutions, the normal unreduced one and that of compensator order reduction, the closed loop time response of the reduced compensator is slightly more oscillatory due to the simplified dynamics of the process that are presented to the compensator.
(a) Time responses to the reference steps for each pool. LQ weights are set at: $R_1 = 1200000$, $R_2 = 600000$ and $R_3 = 240000$, for actuation of gate 1, 2 and 3, respectively.

(b) Position of gate 1 during the simulation.

(c) Position of gate 2 during the simulation.

(d) Position of gate 3 during the simulation.

Figure 49: Results on the experimental canal regarding the centralized LQG controller for 3 gates/3 pools systems without controller order reduction. The controller access all 9 states. The equilibrium configuration throughout the canal is: 614mm, 538mm, 473mm and 315mm, for pool 1, 2 and 3, respectively.
(a) Time responses to the reference steps for each pool. LQ weights are set at: $R_1 = 1200000$, $R_2 = 600000$ and $R_3 = 240000$, for actuation of gate 1, 2 and 3, respectively.

(b) Position of gate 1 during the simulation.

(c) Position of gate 2 during the simulation.

(d) Position of gate 3 during the simulation.

Figure 50: Results on the experimental canal regarding the centralized LQG controller for 3 gates/3 pools systems with controller order reduction to 3 states. The equilibrium configuration throughout the canal is: 663 mm, 559 mm, 493 mm and 320 mm, for pool 1, 2, 3 and 4, respectively.
6.2 Decentralized Control

This section attends experimental results on decentralized and coordinated LQG controllers that are attached to each gate/pool system, therefore, controlling its own water level. So each controller is an independent entity and they achieve a coordinated response by exchanging information about their control strategy with their nearest neighbors. The offtake action is included also by the same principle, that is, each controller accesses the offtake information of its nearest neighboring controllers, so that the control scheme is, in fact, a decentralized scheme. Results shown below only concern 4 gate/4 pool systems, so systems with less number of gates and pools are not tested. Throughout this section, after presenting a reference manipulation example for decentralized controllers, they are analyzed by manipulating the controller design parameters to observe the effects on the experimental canal control, namely: LQ weights parameters, closed loop poles constraint parameter and offtake feedforward action parameter.

Figure 51 shows results of a situation with decentralized LQG controllers. The stabilizing configuration for the gates position used is: 100mm for gates 1, 2 and 3, and 402mm for gate 4, thereby leading to the water level configuration for pools 1, 2, 3 and 4 presented in the description of figure 51. Also for completing the overall equilibrium regime it must be remarked that the intake flow is set at $Q = 28 l/s$. The LQ weights used for the controller design, in which the pole constraint is not considered, are indicated in the description of figure 51(a).

The test attends set point changes in references of all four level, in which they are risen. Reference for pool 1 is the first to be manipulated by increasing its set point by 50mm and afterwards this pool 1 is left to stabilize before making other changes in set points. Thereafter, the same for pool 2, 3 and 4. The time response of each reference step can be seen in figure 51(a) and the gates manipulation by the controllers in figures from 51(b) to 51(e).

The time responses turns out to be relatively faster, although LQ weights are much higher by comparing with the centralized situation (see figure 48). Also, some oscillation is observed due to the unavoidable errors in modeling the process dynamics and the LQ design specifications chosen.
Water levels in pools 1, 2, 3 and 4

(a) Time responses to the reference steps for each pool. LQ weights are set at: $R_1 = 2000000$, $R_2 = 1000000$, $R_3 = 200000$ and $R_4 = 100000$, for actuation of gate 1, 2, 3 and 4, respectively.

(b) Position of gate 1 during the simulation.

(c) Position of gate 2 during the simulation.

(d) Position of gate 3 during the simulation.

(e) Position of gate 4 during the simulation.

Figure 51: Results on the experimental canal regarding the decentralized LQG controllers for 4 gates/4 pools systems. The equilibrium configuration throughout the canal is: 708mm, 622mm, 550mm and 402mm, for pool 1, 2, 3 and 4, respectively.
6.2.1 Manipulating LQ weights

Figures 52 and 53 show experimental results for two different controller designs in which only LQ weights are changed to observe their influence on time responses. In both cases, set points for pools 2, 3 and 4 are kept constant throughout all simulation range and set point for pool 1 is firstly increased by 50mm and then decreased by 100mm. After performing such reference manipulations, the set point is, again, placed in the initial position and the offtake located at pool 1 is opened for 40 minutes at 4l/s and then closed when such time runs out. In the second controller (see figure 53) the offtake 1 is opened at 4.5l/s within the same time interval.

With this test it is possible to see how controller at gate 1 responds to set point changes, how other gates reject this disturbances from the set point change and how all controllers reject the disturbance caused by opening and closing offtake 1.

For running the given tests, the following equilibrium configuration is used for both situations: 100mm for gate 1, 2 and 3 positions, and 400mm for gate 4 position. The intake flow is set at 23.5l/s. This leads to the stationary pool level configuration depicted in the description of figures 52 and 53 for each respective situation.

It is observed that when a lower set of LQ weights is used the responses are faster, both in the reference manipulation case and in the disturbance rejection case through offtake manipulation. Although the ratio between both set of LQ weights is equal to 25, which corresponds to a large increase in LQ weights from one situation to the other, the differences observed are not highly significant. In the next section where the action of the closed loop poles parameter influence is isolated and analyzed, it can be seen that the time response is far more sensitive to this parameter.
Figure 52: Results on the experimental canal regarding the decentralized LQG controllers for 4 gates/4 pools systems with feedforward control. Pool 1 reference and offtake 1 are manipulated, other are kept constant. The equilibrium configuration throughout the canal is: 585 mm, 539 mm, 499 mm and 395 mm, for pool 1, 2, 3 and 4, respectively.
(a) Time responses to the reference steps for each pool. LQ weights are set at: $R_1 = 400000$, $R_2 = 200000$, $R_3 = 400000$ and $R_4 = 200000$, for actuation of gate 1, 2, 3 and 4, respectively. Closed loop poles constraint is used with $\alpha = 1.004$. The offtake action parameter is set at $\xi = 0.6$.

(b) Position of gate 1 during the simulation.

(c) Position of gate 2 during the simulation.

(d) Position of gate 3 during the simulation.

(e) Position of gate 4 during the simulation.

Figure 53: Results on the experimental canal regarding the decentralized LQG controllers for 4 gates/4 pools systems with feedforward control. Pool 1 reference and offtake 1 are manipulated, other are kept constant. The equilibrium configuration throughout the canal is: 591mm, 548mm, 506mm and 392mm, for pool 1, 2, 3 and 4, respectively.
6.2.2 Manipulating closed loop poles constraint action

Figures 54 and 55 present an analysis performed to the time responses of different controller designs based on the manipulation of the closed loop poles constraint parameter. LQ weights and the offtake control action parameter are maintained from one design to the other, in order to isolate pole constraint manipulation effects. Both tests are conducted by only changing pool 1 set points and keeping all others constant. The set point at pool 1 is firstly risen by 50\text{mm} and then dropped by 100\text{mm}. Afterwards, the set point is returned to its starting position and left to stabilize before opening the offtake 1 by 4.5l/s (for both simulations, figure 54 and figure 55) within a time interval of 40 minutes.

These tests are performed under the following equilibrium configuration for the gates position: 100\text{mm} for gates 1, 2 and 3, and 400\text{mm} for gate 4; and an intake flow of 23.5l/s. Initial water levels of all pools over the canal for each simulation are indicated at the description of their respective figures, 54 and 55.

By comparing both simulations, it can be seen that effects on the time response by manipulating the pole constraint parameter are much more felt than in changing LQ weights at the controller design. As expected, when the parameter is increased (figure 54) water levels are driven faster to their respective references, although with some added oscillation due to unmodeled nonlinear effects. Also, the disturbance rejection due to offtake 1 is more effective for higher values of $\alpha$, if the minimization of reference slips immediately after the disturbance occurs are intended in terms of performance.
(a) Time responses to the reference steps for each pool. LQ weights are set at: $R_1 = 2000000$, $R_2 = 1000000$, $R_3 = 200000$ and $R_4 = 100000$, for actuation of gate 1, 2, 3 and 4, respectively. Pole constraint parameter is set at $\alpha = 1.01$. The offtake action parameter is set at $\xi = 0.6$.

(b) Position of gate 1 during the simulation.

(c) Position of gate 2 during the simulation.

(d) Position of gate 3 during the simulation.

(e) Position of gate 4 during the simulation.

Figure 54: Results on the experimental canal regarding the decentralized LQG controllers for 4 gates/4 pools systems with feedforward control. Pool 1 reference and offtake 1 are manipulated, other are kept constant. The equilibrium configuration throughout the canal is: 593mm, 532mm, 499mm and 393mm, for pool 1, 2, 3 and 4, respectively.
(a) Time responses to the reference steps for each pool. LQ weights are set at: \( R_1 = 2000000, R_2 = 1000000, R_3 = 2000000 \) and \( R_4 = 1000000 \), for actuation of gate 1, 2, 3 and 4, respectively. Pole constraint parameter is set at \( \alpha = 1.005 \). The offtake action parameter is set at \( \xi = 0.6 \).

(b) Position of gate 1 during the simulation.

(c) Position of gate 2 during the simulation.

(d) Position of gate 3 during the simulation.

(e) Position of gate 4 during the simulation.

Figure 55: Results on the experimental canal regarding the decentralized LQG controllers for 4 gates/4 pools systems with feedforward control. Pool 1 reference and offtake 1 are manipulated, other are kept constant. The equilibrium configuration throughout the canal is: 594\( \text{mm} \), 543\( \text{mm} \), 504\( \text{mm} \) and 393\( \text{mm} \), for pool 1, 2, 3 and 4, respectively.
6.2.3 Limiting feedforward action

In the current section, experimental results are compared when increasing the feedforward control introduced by accessible offtake readings through a decentralized fashion. A remarked must be done before discussing such results is that, adding all the available feedforward control to the overall control law solution would result in instability of the experimental canal and, so, the parameter used for limiting feedforward action is required. What this parameter does is to weigh the offtake action relatively to other sources of control.

With the purpose of analyzing the key role that feedforward control has on rejecting offtake disturbances, same decentralized controllers are designed and linked to the experimental canal. Using these decentralized controllers, simulations in which feedforward control is regulated by $\xi = 0.6$ (see figure 56) and in which it is regulated by $\xi = 0.3$ (see figure 57) are carried out.

As in previous experimental tests, equilibrium configurations and information on the controllers design are presented in both descriptions of figures 56 and 57. For the case of figure 56 initial gate configuration is as follows: 100mm for gate 1, 2 and 3, and 400mm for gate 4, with an intake flow of 23.5l/s. For the case of figure 57 gate 1, 2 and 3 are set again at 100mm of height but gate 4 is positioned at 388mm and the intake flow is set at 27l/s.

The test presented in figure 56 is conducted by only opening offtake 1 at 4.3l/s and then closing it after 40 minutes. In turn, in figure 57 offtake 1 is opened at 8l/s in the middle of a change in the set point for pool 2, that does not influence the disturbance response, and then closed after 41 minutes. Although tests have their differences, namely a more abrupt disturbance is felt at results from figure 57 since higher water level configurations are used in this case, both experimental simulations are qualitatively comparable. Also, it must be kept in mind that in the second simulation the intake flow is at 27l/s, meaning that there are more 3.5l/s available than the first simulation. If the outcome flow of offtake 1 is analyzed in both cases their difference is approximately equal to the incremented flow at MONOVAR. So this is a reasonable comparison.

As remarked, results with higher $\xi$, that is more feedforward control added to the overall control law, are better than with a weak or even without this kind of control. Although feedforward control can much improve disturbance rejection even in the experimental canal, it must be regulated wisely and with caution.
(a) Time responses to the reference steps for each pool. LQ weights are set at: $R_1 = 2000000, R_2 = 1000000, R_3 = 200000$ and $R_4 = 100000$, for actuation of gate 1, 2, 3 and 4, respectively. Pole constraint parameter is set at $\alpha = 1.004$ and the parameter limiting feedforward action is set at $\xi = 0.6$.

(b) Position of gate 1 during the simulation.

(c) Position of gate 2 during the simulation.

(d) Position of gate 3 during the simulation.

(e) Position of gate 4 during the simulation.

Figure 56: Results on the experimental canal regarding the decentralized LQG controllers for 4 gates/4 pools systems with feedforward control. Pool 1 reference and offtake 1 are manipulated, other are kept constant. The equilibrium configuration throughout the canal is: 587mm, 545mm, 504mm and 393mm, for pool 1, 2, 3 and 4, respectively.
**Figure 57:** Results on the experimental canal regarding the decentralized LQG controllers for 4 gates/4 pools systems with feedforward control. Pool 2 reference and offtake 1 are manipulated, other are kept constant. The equilibrium configuration throughout the canal is: 686\text{mm}, 622\text{mm}, 559\text{mm} and 414\text{mm}, for pool 1, 2, 3 and 4, respectively.
6.3 Fieldwork Remarks

During the tests performed on the experimental canal some problems were faced that affected the results and, in general, the work conducted at NuHCC. Throughout this section such problems are reported not just to identify their influence on final results but also to better understand some implications when the work is brought from the computer to the experimental environment.

6.3.1 Gates dead zones

When linking controllers to the actual canal located at NuHCC gates dead zones were detected, that degrade in much the performance of gate actuation. This dead zones can be due to several factors. Among these are the poor lubrication of the screw actuators and the lack of precision in motors. This limitation implies that a coarse tuning is the only type of gate adjustment available for control, so there are some performance standards that cannot be overtaken since there is no fine tuning.

Figure 58 shows examples on the given limitation. On the left hand side the actual position of gate 1 is overlapped with control commands sent to gate 1. A controller that is taking hold of gate 1 sends a signal to open the gate and afterwards another signal to close. It can be seen that the gates position has a digital behavior and the approximation of the gate position is better when faster commands are sent.

Also, consider the situation in which the controller intends to maintain a stationary pool set point, achieved by sending smoother commands to the gate as the set point is reached. With this limitation what happens is that the controller would be locked in a two step digital range leading the pool level to oscillate over the set point. A illustrative example of such situation is shown on figure 58(b). A controller tries to stabilize a level over a given set point when level is very close to the set point. The gate commands over the actual gate position are presented in figure 58(b). It can be seen that the actual gate position keeps jumping between 87mm, 93mm and 99mm and the controller tries to hopelessly sustain the level over the set point.

6.3.2 Problems faced

Apart from gates dead zones discussed before there were some problems faced when testing controllers at the experimental canal. Below is shown a list of such issues:

- Shortage of water and many leakages throughout the canal have led to many canceled tests that were scheduled and little water available at the tanks which, in turn, led to lower allowable intake flows;
- A new sensor calibration was required for gate 4 due to malfunctions at the local PLC;
- Some malfunctions at the last gate with problems at the position sensor and in sending commands to its actuator. The sensor malfunction led the gate to move around even when reading its own noise, that is, without sending any commands at all;
- Pool 4 drained water from underneath gate 4, that is an overshoot gate;

![Figure 58: Illustrative examples of gates dead zones.](image)

(a) Time evolution of the gate 1 current position (b) Controller stabilizing a water pool level over a stationary reference.
As sensors used to read water levels at each pool are potentiometers linked to a float with a caster, the cable was always jumping off the caster.

7 Conclusions

Não tive tempo de fazer a conclusão mas enquanto revê a tese eu faço a conclusão e depois envio-lha por e-mail.

References


A LQ control with accessible disturbances and neighboring agent coordination

Below is the solution for the decentralized LQ control with offtake action approach, presented in this work for MIMO systems situation. Here is not considered the reference accommodation through integral effect that was used together with the LQ controllers, so, the Riccati solution appears in a decentralized form, as explained in section 3.1. Therefore, it is possible to compute the solution for the decentralized LQ problem using a generalized matrix form and computing the solution for all agents at a time, as done below.

The solution for the problem referred in section 3.1 involves applying Pontryagin’s Minimum principle from appendix B. The performance index for the LQ problem is given by equation (16). In this special case, hamiltonian is given by,

\[ H(k) = \lambda^T(k+1)[Ax(k) + Bu(k) + \Psi_{\text{gate}}(k) + \Gamma_{\text{off}}(k)] \]

\[ - \frac{1}{2}(x^T(k)C^T Cx(k) + u^T(k)Ru(k)) \]

since \( f(u(k), x(k), k) \) represents the right hand side of the state equation and Lagrangian function is,

\[ L(k) = -\frac{1}{2}[x^T(k)C^T Cx(k) + u^T(k)Ru(k)] \]

In (54), \( \lambda \) is the co-state variable, \( \Psi \) has information about local agent interactions and \( \Gamma \) information relatively to disturbance action. Stationary condition (80) is given by,

\[ \frac{\partial H(k)}{\partial u(k)} = \lambda^T(k+1)B - Ru(k) = 0 \]

which produces the optimal control expression:

\[ u_{opt}(k) = R^{-1}B^T \lambda(k+1) \]  

In order to detach state and co-state into two distinct equations, express the optimal control law as a function of measured variables and integrate distributed coordination, we consider that the solution for co-state is in its usual form as in typical LQ problems plus two terms, one responsible for disturbances feedforwarding (\( g_{\text{off}} \)) and other for coordination (\( g_{\text{gate}} \)). It is assumed that there are a matrix \( P'[n \times n] \), in which \( n \) is the number of total states of the global MIMO system, and vectors \( g_{\text{gate}} \) and \( g_{\text{off}} \) both \([m \times 1]\), such that, for every \( k \),

\[ \lambda(k) = -Px(k) + g_{\text{gate}} + g_{\text{off}} \]  

Here, \( g_{\text{gate}} \) and \( g_{\text{off}} \) are taken apart for simplification purposes, but they could very well be merged into one. For \( P \), \( g_{\text{gate}} \) and \( g_{\text{off}} \) to be constant it is assumed that \( N \to \infty \). Combining (57) with (58) yields,

\[ u_{opt}(k) = -R^{-1}B^TPx(k+1) + R^{-1}B^T g_{\text{gate}} + R^{-1}B^T g_{\text{off}} \]

which requires the next time instant for the state, \( x(k+1) \). This can be solved by using state equation presented in section 3.1 and it goes as follows,

\[ u(k) = -R^{-1}B^TP(Ax(k) + Bu(k) + \Psi_{\text{gate}}(k) + \Gamma_{\text{off}}(k)) + R^{-1}B^T g_{\text{gate}} + R^{-1}B^T g_{\text{off}} \]

Solving with respect to \( u(k) \) yields,

\[ (I + R^{-1}B^TPB)u(k) = -R^{-1}B^TPAx(k) + R^{-1}B^T (g_{\text{gate}} - P\Psi_{\text{gate}}(k)) + R^{-1}B^T (g_{\text{off}} - P\Gamma_{\text{off}}(k)) \]

76
where \( I \) denotes an identity matrix. This can be arranged in,

\[
\begin{align*}
\hat{u}_{\text{gate}}(k) &= - (I + R^{-1}B^TPB)^{-1}R^{-1}B^TPAx(k) \\
&\quad + (I + R^{-1}B^TPB)^{-1}R^{-1}B^T(g_{\text{gate}} - P\Psi d_{\text{gate}}(k)) \\
&\quad + (I + R^{-1}B^TPB)^{-1}R^{-1}B^T(g_{\text{off}} - P\Gamma d_{\text{off}}(k))
\end{align*}
\]

Then, three pieces of (62) are immediately highlighted. The one that is multiplied by the state is, typically, the solution for LQ problems assuming an infinity horizon and will be called \( K_{\text{fb}} \). Other two are the coordination and feedforward control solutions. Therefore, this can be rewritten in,

\[
\hat{u}_{\text{gate}}(k) = -K_{\text{fb}}x(k) + \hat{u}_{\text{gate}}(k) + \hat{u}_{\text{off}}(k)
\]

being that the optimal LQ gain, \( K_{\text{fb}} \), is

\[
K_{\text{fb}} = (I + R^{-1}B^TPB)^{-1}R^{-1}B^TPA
\]

the decentralized control,

\[
\hat{u}_{\text{gate}}(k) = (I + R^{-1}B^TPB)^{-1}R^{-1}B^T(g_{\text{gate}} - P\Psi d_{\text{gate}}(k))
\]

and the feedforward control action \( \hat{u}_{\text{off}} \),

\[
\hat{u}_{\text{off}}(k) = (I + R^{-1}B^TPB)^{-1}R^{-1}B^T(g_{\text{off}} - P\Gamma d_{\text{off}}(k))
\]

Nevertheless matrix \( P \) and vectors \( g_{\text{gate}} \) and \( g_{\text{off}} \) are still unknown. For this purpose the adjoint function (79) is used. Partial derivative of the Lagrangian function (55) with respect to \( x(k) \) is

\[
\frac{\partial L(k)}{\partial x(k)} = -(Cx(k))C
\]

The adjoint equation is thus,

\[
\lambda(k) = A^T\lambda(k+1) + C^TCx(k)
\]

In the next step, equation for the state in closed loop form is needed. This can be obtained by, firstly, combining the optimal control law (57) with co-state equation (58) and, then, applying the result to the open loop state equation presented in section 3.1. Thus,

\[
x(k+1) = Ax(k) + BR^{-1}B^TPx(k+1) + g_{\text{gate}} + g_{\text{off}}] + \Psi d_{\text{gate}}(k) + \Gamma d_{\text{off}}(k)
\]

Solving with respect to \( x(k+1) \) produces the final closed loop state equation,

\[
x(k+1) = [I + BR^{-1}B^TP]^{-1}Ax(k) + [I + BR^{-1}B^TP]^{-1}BR^{-1}B^Tg_{\text{gate}}
\]

\[
\quad + [I + BR^{-1}B^TP]^{-1}BR^{-1}B^Tg_{\text{off}} + [I + BR^{-1}B^TP]^{-1}\Psi d_{\text{gate}}(k)
\]

\[
\quad + [I + BR^{-1}B^TP]^{-1}\Gamma d_{\text{off}}(k)
\]

where this time \( I \) is an identity matrix by blocks since each output has a certain number of states.

Pick up the co-state solution and feed it in both sides into the adjoint equation (68) to get,

\[
-Px(k) + g_{\text{gate}} + g_{\text{off}} = -A^TPx(k+1) + A^Tg_{\text{gate}} + A^Tg_{\text{off}} - C^TCx(k)
\]

Then, with closed loop state equation (70), transform equation (71) such that it only depends on \( x(k), g_{\text{gate}} \) and \( g_{\text{off}} \). By isolating all \( x(k) \) terms, we get,

\[
\begin{align*}
\{ P - A^T[I + BR^{-1}B^TP]^{-1}A - C^TC \} x(k) \\
\quad - \{ I + A^T[I + BR^{-1}B^TP]^{-1}BR^{-1}B^T - A^T \} (g_{\text{gate}} + g_{\text{off}}) \\
\quad - A^TP[I + BR^{-1}B^TP]^{-1}\Psi d_{\text{gate}}(k) - A^TP[I + BR^{-1}B^TP]^{-1}\Gamma d_{\text{off}}(k) = 0
\end{align*}
\]

77
For equation (72) to be true for all $x$, $d_{\text{gate}}$ and $d_{\text{off}}$, $P$, $g_{\text{gate}}$ and $g_{\text{off}}$ must satisfy simultaneously the discrete-time algebraic Riccati equation,

$$P = A^T P \left[I + BR^{-1}B^T P \right]^{-1} A + C^T C$$

(73)

and the condition on combination of vectors $g_{\text{gate}}$ and $g_{\text{off}}$:

$$\{I + A^T P[I + BR^{-1}B^T P]^{-1}B^T - A^T\} \left(g_{\text{gate}} + g_{\text{off}}\right) =$$

$$= -A^T P [I + BR^{-1}B^T P]^{-1} \Psi_{d_{\text{gate}}}(k) - A^T P [I + BR^{-1}B^T P]^{-1} \Gamma_{d_{\text{off}}}(k)$$

(74)

For equation (74), it is still possible to decouple each vector $g$ and achieve more subtle expressions for them,

$$\begin{cases} g_{\text{gate}} = - \{I + A^T P[I + BR^{-1}B^T P]^{-1}B^T - A^T\}^{-1} A^T P[I + BR^{-1}B^T P]^{-1} \Psi_{d_{\text{gate}}}(k) \\ g_{\text{off}} = - \{I + A^T P[I + BR^{-1}B^T P]^{-1}B^T - A^T\}^{-1} A^T P[I + BR^{-1}B^T P]^{-1} \Gamma_{d_{\text{off}}}(k) \end{cases}$$

(75)
B Pontryagin’s Minimum Principle in discrete time

Consider the discrete time plant modeled by the nonlinear equation,

\[ x(k + 1) = f(x(k), u(k), k) \]  

(76)

with a given initial condition \( x(0) \) and where \( k \) stands for discrete-time, \( u \in \mathbb{R}^m \) is the input for the plant, \( x \in \mathbb{R}^n \) the state and \( f : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n \) is a vector function which describes plant dynamics. Also consider the next performance index to be minimized within the plant being controlled,

\[ J(u) = \Phi(x(N)) + \sum_{k=0}^{N-1} L(x(k), u(k), k) \]  

(77)

where \( L \) is the Lagrangian function. Then Hamiltonian function is given by

\[ H(k) = \lambda^T(k + 1) f(x(k), u(k), k) + L(x(k), u(k), k) \]  

(78)

where \( \lambda \) is the co-state.

It is assumed that \( N \to \infty \), so there are no constraints either on the final state \( x(N) \) or on \( u(k) \).

According to the Pontryagin Minimum Principle in discrete time, along an optimal trajectory, the state \( u \), the co-state \( \lambda \) and the optimal control \( u \) satisfy the following set of conditions:

a) State equation (76) with the specified initial condition;

b) Adjoint equation

\[ \lambda(k) = \left( \frac{\partial f(x(k), u(k), k)}{\partial x(k)} \right)^T + \left( \frac{\partial L(x(k), u(k), k)}{\partial x(k)} \right)^T \]  

(79)

c) Stationarity condition

\[ \frac{\partial H(k)}{\partial u(k)} = 0 \]  

(80)

d) Co-state terminal condition

\[ \lambda(N) = \frac{\partial \Phi}{\partial x(N)} \]  

(81)
C Centralized Identification Results

Below are presented identification results for centralized control design. Since LQG controllers were made for all possible situations, results for each one of them are shown. Results are shown in the form of cross correlation of residue which is a more rigorous way of analyzing and comparing them, relatively to just calculate the output fit percentage. For residue it is meant the difference between process output and model output identified with the same data from the previous. Attached to the cross correlation is its 99% confidence interval, represented as an yellow box.

Input signals that were used to identify these systems are binary signals fluctuating around a certain offset value and slow enough to neglect nonlinearities in the process. No signal processing was done since noise presented in the data collected is due to numerical effects within the simulation model. In physical situations these effects do not occur, so they can be ignored.

All these results come from simulation model [8]. Comparisons between ARX models obtained and final state-space models used in controllers are shown but only for 1 gate controller, 2 gates controller and 3 gates controller. For the 4 gates centralized controller are only shown results for state-space structure to not overextend the results section.

C.1 One Gate - One Pool

Figures 59(a) and 60(a) represent the model output and its comparison with the process for ARX and state-space model structure, respectively. Percentage of fit is shown on the right corner. Figures 59(b) and 60(b) show residue analysis for both, ARX and state-space representations.

In this situation only gate 3 was manipulated and only pool 3 was measured, all other gates were kept constant.

Figure 59: Residue analysis and fitting for ARX structure presented in equation (8).

Figure 60: Residue analysis and fitting for state-space structure.
C.2 Two Gates - Two Pools

In this section, middle gates and pools are manipulated and measured, that is, nr 2 and nr 3. Figure 61 shows cross correlation of residue between process output and ARX linear model output. Top figures within the previous illustrate residue analysis for both possible channels, gate 2 to pool 2 and gate 3 to pool 2. Bottom figures show the residue analysis for channels, gate 2 to pool 3 and gate 3 to pool 3. The same description is applied to figure 62 but in this case, the analysis was performed to the state-space structure.

Figure 61: Residue analysis for ARX structure. Gates 2 and 3 as well as pools 2 and 3 were used.

Figure 62: Residue analysis for state-space structure. Gates 2 and 3 as well as pools 2 and 3 were used.
C.3 Three Gates - Three Pools

Here gates 1, 2 and 3 were manipulated and pools 1, 2 and 3 were measured. Figures 63(a), 63(b) and 63(c) show cross correlation of residue for pools 1, 2 and 3, between process output and ARX model output. To each pool is associated three gates, which makes three possible channels to analyze, and so, three graphics are observed. Figure 64 is similar to 63 but in this case, the analysis was performed to the state-space structure.

Figure 63: Residue analysis for ARX structure. Gates 1, 2 and 3 as well as pools 1, 2 and 3 were used.
Figure 64: Residue analysis for state-space structure. Gates 1, 2 and 3 as well as pools 1, 2 and 3 were used.
C.4 Four Gates - Four Pools

In this section, figures 65 to 68 show a residue analysis relatively to each pool. Since there is a 4 gate/4 pool system to model, number of possible analyzable channels increase to 16. So, each figure has 4 cross correlations, each between the process output of the respective pool and the linear model output for that same pool, when it is excited by a given gate.

In these figures only state-space structure is used.

Figure 65: Residue analysis accounting influences of all gates in water level on pool 1 through state-space structure.

Figure 66: Residue analysis accounting influences of all gates in water level on pool 2 through state-space structure.
Figure 67: Residue analysis accounting influences of all gates in water level on pool 3 through state-space structure.

Figure 68: Residue analysis accounting influences of all gates in water level on pool 4 through state-space structure.
D Decentralized Identification Results

Figures 69 to 72 show cross correlation for each output, being that, in each figure are observed all possible channels between inputs and the respective output. Only global decentralized MIMO is analyzed which uses a state-space structure.

Figure 69: Residue analysis accounting influences of all gates in water level on pool 1 for decentralized case through state-space structure.

Figure 70: Residue analysis accounting influences of all gates in water level on pool 2 for decentralized case through state-space structure.
Figure 71: Residue analysis accounting influences of all gates in water level on pool 3 for decentralized case through state-space structure.

Figure 72: Residue analysis accounting influences of all gates in water level on pool 4 for decentralized case through state-space structure.
E Other Experimental Results

In this section other experimental tests done with two and three gate centralized controllers that are not included on the report body, are shown below. Also, some decentralized controllers with and without feedforward control tests are presented.

In these tests both reference set points, offtakes and inclusively MONOVAR are manipulated for observing the behavior of controllers when applied to an experimental canal.

In the descriptions of the figures showing experimental results, all information is presented about the initial gates position, initial pool levels configuration as well as specifications used in controller designs.

E.1 Two Gate Centralized LQG Controller

Figure 73 shows a centralized LQG controller applied to 2 gates/2 pools systems. Three set point changes in both pools controlled, pool 2 and 3 are made with both delayed in order to observe their individual time response. By observing the figure a situation must be remarked. It is the fact that set point of pool 2 goes below the set point of pool 3, which makes the level in 3 to be driven out of its reference and to follow the level at pool 2. The issue is solved as soon as the set point of pool 3 tells the controller to drop the level, thereby, recovering the control of the pool level. Figure 74 shows a centralized LQG controller applied to 2 gates/2 pools systems when controlling two stationary set points for pool 2 and 3.
(a) Time responses to the reference steps for each pool. LQ weights are set at: $R_2 = 500000$ and $R_3 = 5000000$, for actuation of gate 2 and 3, respectively.

(b) Position of Gate 2 during the simulation.  

(c) Position of Gate 3 during the simulation.

Figure 73: Results on the experimental canal regarding the centralized LQG controller for 2 gates/2 pools systems. The equilibrium configuration throughout the canal is: 708mm, 608mm, 530mm and 339mm, for pool 1, 2, 3 and 4, respectively. The gates positions used are: 100mm for gates 1, 2 and 3, and 400mm for gate 4. The intake flow is set at 28l/s.
(a) Reference stabilization for each pool. LQ weights are set at: \( R_2 = 500000 \) and \( R_3 = 5000000 \), for actuation of gate 2 and 3, respectively.

(b) Position of Gate 2 during the simulation.

(c) Position of Gate 3 during the simulation.

Figure 74: Results on the experimental canal regarding the centralized LQG controller for 2 gates/2 pools systems. The references are kept constant throughout all simulation. The equilibrium configuration throughout the canal is: 567\( \text{mm} \), 518\( \text{mm} \), 489\( \text{mm} \) and 362\( \text{mm} \), for pool 1, 2, 3 and 4, respectively. The gates positions used are: 100\( \text{mm} \) for gates 1, 2 and 3, and 420\( \text{mm} \) for gate 4. The intake flow is set at 20\( l/s \).
E.2 Three Gate Centralized LQG Controller

Results with centralized LQG controller for 3 gates/3 pools systems are observed in figures 75 and 76. The test at the figure 75 focuses on MONOVAR and offtake 1 manipulation, in which MONOVAR is opened from 27l/s to 35l/s and offtake 1 is opened with an outcome flow of 12.5l/s at 180 minutes until the end of the simulation. Figure 76 shows reference manipulations in all three pools, pool 1, 2 and 3. The set point of pool 1 is firstly risen giving the controller enough time to stabilize the three pools and afterwards the same is done to the next two pools.
Figure 75: Results on the experimental canal regarding the centralized LQG controller for 3 gates/3 pools systems. The references are kept constant throughout all simulation and MONOVAR valve and offtake 1 are manipulated. The equilibrium configuration throughout the canal is: 629mm, 550mm, 491mm and 316mm, for pool 1, 2, 3 and 4, respectively. The gates positions used are: 100mm for gates 1, 2 and 3, and 350mm for gate 4. The intake flow is set at 27l/s.
Figure 76: Results on the experimental canal regarding the centralized LQG controller for 3 gates/3 pools systems. The equilibrium configuration throughout the canal is: \(654 \text{mm}, 574 \text{mm}, 519 \text{mm} \) and \(287 \text{mm}\), for pool 1, 2, 3 and 4, respectively. The gates positions used are: \(100 \text{mm}\) for gates 1, 2 and 3, and \(300 \text{mm}\) for gate 4. The intake flow is set at \(26.5 \text{l/s}\).
E.3 Four Gate Decentralized LQG Controllers

Figure 77 shows results regarding decentralized controllers without feedforward control applied to all pools throughout the canal. In this test, MONOVAR valve is opened at the beginning of the simulation from 28l/s to 35l/s, followed by brief offtake openings at offtakes 1, 2, 3 and 4 and they are opened with an outcome flow of 15.4l/s, 12l/s, 8.2l/s and 13.6l/s, respectively, by this same sequence. The disturbance rejections at each pool as well as the behavior of controllers over all time range can be observed through the figure.
(a) Reference stabilization for each pool. LQ weights are set at: $R_1 = 2000000$, $R_2 = 1000000$, $R_3 = 200000$ and $R_4 = 100000$, for actuation of gate 1, 2, 3 and 4, respectively.

(b) Position of Gate 1 during the simulation.

(c) Position of Gate 2 during the simulation.

(d) Position of Gate 3 during the simulation.

(e) Position of Gate 4 during the simulation.

Figure 77: Results on the experimental canal regarding the decentralized LQG controller for 4 gates/4 pools systems. References are kept constant and all offtakes are manipulated. The equilibrium configuration throughout the canal is: 723mm, 613mm, 539mm and 407mm, for pool 1, 2, 3 and 4, respectively. The gates positions used are: 100mm for gates 1, 2 and 3, and 395mm for gate 4. The intake flow is set at 28l/s.
E.4 Four Gate Decentralized LQG Controllers with Feedforward Control

Figure 78 shows results regarding decentralized controllers with feedforward control applied to all pools throughout the canal. In this test, MONOVAR valve is opened at the beginning of the simulation from 28 l/s to 35 l/s, but this time it is followed by brief offtake openings at offtakes 4, 1 and 2. Offtake 3 is not manipulated. They are opened with an outcome flow of 6.8 l/s, 13.7 l/s and 12.1 l/s, respectively, by this same sequence. The disturbance rejections at each pool as well as the behavior of controllers over all time range can be observed through the figure.
(a) Reference stabilization for each pool. LQ weights are set at: $R_1 = 2000000$, $R_2 = 1000000$, $R_3 = 200000$ and $R_4 = 100000$, for actuation of gate 1, 2, 3 and 4, respectively. The feedforward and the pole constraint parameters are set at $\xi = 0.5$ and $\alpha = 1.004$.

(b) Position of Gate 1 during the simulation.

(c) Position of Gate 2 during the simulation.

(d) Position of Gate 3 during the simulation.

(e) Position of Gate 4 during the simulation.

Figure 78: Results on the experimental canal regarding the decentralized LQG controller for 4 gates/4 pools systems with feedforward control and closed loop pole constant. References are kept constant and offtakes 1, 2 and 4 are manipulated. The equilibrium configuration throughout the canal is: 714mm, 625mm, 567mm and 400mm, for pool 1, 2, 3 and 4, respectively. The gates positions used are: 100mm for gates 1, 2 and 3, and 400mm for gate 4. The intake flow is set at 28l/s.