Design of a Fault Tolerant Control System for a Water Delivery Canal
— Centralized multivariable control with actuator faults

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1 Introduction

Freshwater scarcity has become a major concern throughout the world. This is mainly due to the significant rise in water consumption that has been verified, where irrigation systems assume great responsibility. These systems entail a large social-economic impact since a high percentage of the Gross Domestic Product of several countries stems from agricultural irrigation products.

In irrigation systems, the number one difficulty is the management of the available water, which may lead to significant waste if not attended carefully. It is therefore important to improve control systems of irrigation channels in order to minimize these losses. Since water demand for irrigation varies with time, we seek a robust and stable control system that is able to respond quickly enough to sudden variations in the water levels, so that those can be maintained at the desired values. Even though good results have already been achieved in this matter, physical problems, such as actuator or sensor failure, will most likely destabilize the regular operation of the controlled system and lead to unstable behaviors that may result in substantially more waste. For this reason, it is essential to develop fault tolerant control systems.

In this report, we only consider actuator faults, i.e., when a gate does not move, regardless of the command received by the control system. Assuming that the actuator of gate 2 fails, the objective is to design a controller based on the other pools and respective gates, so that the system does not become unstable.

On this basis, we begin by creating a linear model of the canal through identification of the existent nonlinear one. Once we have a linear model that describes the canal, a Linear-Quadratic Gaussian controller can be designed to control the canal in regular operation. So as to deal with actuator faults, after the nonlinear model has been identified for the case that gate 2 gets stuck at a fixed position, a second controller is designed based on the resultant linear model.

The report is organized as follows: Section 2 gives a brief description of the experimental canal. System identification is discussed in section 3, where after different possible methods are presented and compared for the single-input, single-output (SISO) case, one method is chosen and the results presented for the multiple-input, multiple-output (MIMO) case. In section 4 we study the Linear-Quadratic Gaussian controller and in section 5 the influence of its parameters on the system response. Also in section 5 results of the controlled nonlinear MIMO system are shown. A control system tolerant to faults in gate 2 is developed in section 6, where results of the controlled nonlinear MIMO system with a switch between controllers are also shown. In section 7 experimental results are presented. Conclusions are drawn in section 8.
2 Canal Description

The canal on which this study is based belongs to the Hydraulics and Canal Control Center (NuHCC) of the University of Évora, Portugal. This system has an automatic canal and a return canal (traditional canal). The automatic canal consists of four pools separated from each other by undershot gates, whereas the last pool is terminated by an overshot one, as shown in figure 1. The traditional canal closes the circuit by returning the water drained by gate 4 to a reservoir placed upstream of the automatic canal.

![Figure 1: Schematic of the NuHCC automatic canal.](image)

The automatic canal is 141 m long and has a trapezoidal shape, with a cross-section of bottom width 0.15 m, side slope 1:0.15 (V:H) and depth 0.90 m. Its average longitudinal bottom slope is about $1.5 \times 10^{-3}$ and the maximum flow admitted is 9 l/s.

Three level sensor are placed along each pool, measuring the corresponding upstream, center and downstream water levels, which in figure 1 are represented by $M_i$, $C_i$ and $J_i$, respectively. In the same figure, gate positions $U_i$ represent the manipulated variables and $Q_i$ the flows of the offtakes (lateral holes near the bottom of the canal), which are controlled by the motorized butterfly valves $V_{oi}$. We will only use the downstream level measures of each pool, that is $J_i$.

3 System identification

In order to design a control system, a model that describes the relevant dynamics of the canal is needed. The model can be attained either by using the St. Venant equations or by using system identification models based on operational data from the canal. A nonlinear model based on the St. Venant Equations has already been developed for the present irrigation canal. From this nonlinear model, a linear one can be built based on parametric methods that estimate parameters or model structures such as

- Autoregressive with exogenous inputs (ARX)
- Autoregressive Moving Average with exogenous inputs (ARMAX)
- Box-Jenkins (BJ)
- Output-Error (OE).

Previous studies on this same canal proved ARX to be the best choice [4], [1]. Since noise attached to the data collected from simulation of the canal nonlinear model is not white [4], ARX would not be the most appropriate structure if the ultimate goal was identification. As the aim is to obtain a suitable control system and the other models do not improve much the results when compared to the ones obtained with ARX, the latter is preferable for it is the simplest one.

Once the method to be used is selected, the next step is to choose an appropriate signal to excite the system, or in this case the nonlinear model created. The Pseudo-Random Binary Sequence (PRBS) was selected, as it excites the plant over a wider range of frequencies than most other signals, allowing a better identification of system transients. Furthermore, the signal amplitude must be diminished so as to avoid nonlinearities, its period must be large enough for the water level to stabilize and, since we are dealing with a slow process, the sampling period must also be large. It has been seen in previous studies [4] that the best option is 2s.

Although in those same studies the intake flow was set to 2l/s, it was also observed that increasing the intake flow made the system response faster. For that reason, the value of 3l/s was chosen for the present study.

3.1 SISO system identification

In the single-input, single-output case, only pool 1 is taken into account, the input being the position of gate 1 and the output the downstream level of pool 1. In order to obtain data to identify the system, a PRBS is applied to the input (position of gate 1) around a set point, while the other gate positions are kept constant also at the set points. The set points around which the system is to be operated are 70 cm, 60 cm, 50 cm and 40 cm, for the water levels of pools from 1 to 4, respectively. The respective gate positions that lead to those levels are 5.96 cm, 6.01 cm, 6.09 cm and 28.77 cm. The data collected from the simulation of the nonlinear model is shown in figure 2. We can observe that when the gate opens (gate position assumes a higher value), the water level decreases, and the other way around, which makes sense since the gate is placed at the end of the pool.

Identification using an ARX model is based on the computation of least squares estimates, which can be attained by solving the equation

\[ A(q^{-1}) y(t) = B(q^{-1}) u(t - n_k) + e(t), \]  

(1)
where $q^{-1}$ represents the backward shift operator, $n_k$ the time delay inherent to the systems and

$$A(q^{-1}) = 1 + a_1 q^{-1} + \ldots + a_{n_a} q^{-n_n},$$

$$B(q^{-1}) = b_0 + b_1 q^{-1} + \ldots + b_{n_b} q^{-n_b},$$

in which $a_i, i = 1, \ldots, n_a$ and $b_i, i = 0, \ldots, n_b$ are real parameters, with $n_a$, $n_b$, positive integers.

The aim of the identification process is to find the parameters $a_i$ and $b_i$, given the orders of the polynomials $A(q^{-1})$ and $B(q^{-1})$, $n_a$ and $n_b$, and the delay $n_k$. However, since the least squares method admits data to have zero mean, before the identification process the mean needs to be removed from the data, as well as the initial transient, which is evident in figure 2. The best orders found, i.e., the orders that lead to the best fit between the data and the model response are $n_a = 4$ and $n_b = 2$ and the delay $n_k = 1$. So that the adjustment of the model response to the data can be quantified, we define a measure of fit given by

$$fit(\%) = \frac{\|y_{data} - y_{model}\|}{\|y_{data} - \bar{y}_{data}\|} \times 100,$$

where $y_{data}$ is the output of the validation data, $\bar{y}_{data}$ is the mean of the output of the validation data and $y_{model}$ is the model output.

Figure 2: Open-loop response of pool 1 in the nonlinear model, when a PRBS is applied to the input around an equilibrium value of 5.96 cm.
Comparison between data and model

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{comparison.png}
\caption{Comparison between the data collected from the open-loop simulation of the nonlinear model and the response of the ARX model, without filtering the data.}
\end{figure}

For this case a fit of 94.22\% was obtained and the comparison between the data and the model response is presented in figure 3.

Since ARX assumes the data noise to be white, when in fact it is not, it is expected that the reduction of noise improves the previous value. With this in mind, a butterworth filter is applied to the data before the identification. Figure 4 shows the fit between the filtered data and the new model, which was improved in 4\%. The orders that lead to this result are \( n_a = 4 \) and \( n_b = 4 \), with a delay of \( n_k = 1 \).

On the other hand, if an ARMAX structure is used, the result obtained for the fit between data and model is barely improved. As opposed to the least squares method, the maximum likelihood method provides unbiased estimates in the presence of colored noise. With respect to the ARX model, the ARMAX model has an extra \( C(q^{-1}) \) polynomial in the equation and the latter is given by

\[ A(q^{-1}) y(t) = B(q^{-1}) u(t - n_k) + C(q^{-1}) e(t), \quad (2) \]

where \( q^{-1} \) represents the backward shift operator, \( n_k \) the time delay inherent to the systems and

\[ C(q^{-1}) = 1 + c_1 q^{-1} + \ldots + c_{n_c} q^{-n_c}, \]

in which \( c_i, i = 1, \ldots, n_c \) is a real parameter, with \( n_c \) a positive integer.
Comparison between data and model

![Comparison between data and model](image)

Figure 4: Comparison between the data collected from the open-loop simulation of the nonlinear model and the response of the ARX model, after filtering the data.

The results obtained with and without the use of the filter are presented in figures 6 and 5, respectively. The fit was improved in less than 0.3%.

### 3.2 MIMO system identification

In the multiple-input, multiple-output case, only pools 1, 2 and 3 are taken into account, as a matter of simplicity, since the fourth pool is ended by an overshot gate. As seen in section 3.1, ARX is the best option if the data is filtered before submitted to the identification process. The equation solved by ARX in the MIMO form is

\[
A(q^{-1})y(t) = B(q^{-1})u(t) + e(t),
\]

where \( e(t) \) is a Gaussian white noise sequence, \( y \in \mathbb{R}^{n_y} \), with \( n_y \) the number of outputs, \( u \in \mathbb{R}^{n_u} \), with \( n_u \) the number of inputs, \( A(q^{-1}) \in \mathbb{R}^{n_y \times n_y} \) and \( B(q^{-1}) \in \mathbb{R}^{n_y \times n_u} \). Each entry of matrix \( A(q^{-1}) \) is a polynomial such that the \( m,l \) element, with \( m,l = 1,\ldots, n_y \), is given by

\[
a_{ml}(q^{-1}) = 1 + \sum_{i=1}^{n_{a_{ml}}} \alpha_{ml}^i q^{-i}
\]

in that \( \alpha_{ml}^i \in \mathbb{R} \) and \( n_{a_{ml}} \) are positive integers that represent the orders of the polynomials. Similarly, each entry of matrix \( B(q^{-1}) \) is a polynomial such
Figure 5: Comparison between the data collected from the open-loop simulation of the nonlinear model and the response of the ARMAX model, without filtering the data.

Figure 6: Comparison between the data collected from the open-loop simulation of the nonlinear model and the response of the ARMAX model, after filtering the data.
that the m-j element, with \( j = 1, \ldots, n_u \), is given by

\[
b_{mj}(q^{-1}) = \sum_{i=0}^{n_{kmj}} \beta^i_{mj} q^{-n_{kmj}-i},
\]

where \( \beta^i_{mj} \in \mathbb{R} \), \( n_{kmj} \) are positive integers that represent the delays inherent to the system and \( n_{kmj} \) are positive integers that represent the orders of the polynomials. In order to match the distributed control algorithm to be used, matrix \( A(q^{-1}) \) is considered to have the structure

\[
A(q^{-1}) = \begin{bmatrix}
1 + \sum_{i=1}^{n_{a11}} \alpha^i_{11} q^{-i} & 0 & 0 \\
0 & 1 + \sum_{i=1}^{n_{a22}} \alpha^i_{22} q^{-i} & 0 \\
0 & 0 & 1 + \sum_{i=1}^{n_{a33}} \alpha^i_{33} q^{-i}
\end{bmatrix}.
\]

This means that the water level of one pool is not influenced by the water levels in the other pools. Instead, the values of each water level only depend on the values of the same water level in previous time instants. In the same way, it also happens that not all inputs influence each water level. In fact, the position of gate 3 barely affects the water level of pool 1 and a change in the position of gate 1 is barely noticed by the water level of pool 3. Therefore, the structure of matrix \( B(q^{-1}) \), whose m-j entry gives the influence of input j on output m, becomes

\[
B(q^{-1}) = \begin{bmatrix}
\sum_{i=0}^{n_{b11}} \beta^i_{11} q^{-n_{k11}-i} & \sum_{i=0}^{n_{b12}} \beta^i_{12} q^{-n_{k12}-i} & 0 \\
\sum_{i=0}^{n_{b21}} \beta^i_{21} q^{-n_{k21}-i} & \sum_{i=0}^{n_{b22}} \beta^i_{22} q^{-n_{k22}-i} & \sum_{i=0}^{n_{b23}} \beta^i_{23} q^{-n_{k23}-i} \\
0 & \sum_{i=0}^{n_{b32}} \beta^i_{32} q^{-n_{k32}-i} & \sum_{i=0}^{n_{b33}} \beta^i_{33} q^{-n_{k33}-i}
\end{bmatrix}.
\]

In order to find an ARX model that approximates the system, the orders of the polynomials, \( n_{a_{ml}} \) and \( n_{kmj} \), and the delays \( n_{kmj} \) must be chosen. These orders and delays can be organized into three matrices that have the same structure as matrices \( A(q^{-1}) \) and \( B(q^{-1}) \). Those matrices can be written as

\[
N_A = \begin{bmatrix}
n_{a11} & 0 & 0 \\
0 & n_{a22} & 0 \\
0 & 0 & n_{a33}
\end{bmatrix},
N_B = \begin{bmatrix}
n_{b11} & n_{b12} & 0 \\
n_{b21} & n_{b22} & n_{b23} \\
0 & n_{b32} & n_{b33}
\end{bmatrix},
N_K = \begin{bmatrix}
n_{k11} & n_{k12} & 0 \\
n_{k21} & n_{k22} & n_{k23} \\
0 & n_{k32} & n_{k33}
\end{bmatrix}.
\]

So as to collect data for the identification, the simulink model of the canal is simulated. In the simulation, three PRBS are applied to inputs (gate positions) 1, 2 and 3 around the equilibrium values of 5.96 cm, 6.01 cm and 6.09 cm, respectively, while the fourth gate is kept at the position 28.77 cm. The data obtained is shown in figures 7 (water levels) and 8 (gate positions).
Figure 7: Open-loop response of pools 1, 2 and 3 when a PRBS is applied to each gate position around the equilibrium values 5.96 cm, 6.01 cm and 6.09 cm, for gates 1, 2 and 3, respectively, while the fourth gate is kept at the position 28.77 cm.

However, the identification results obtained were unsatisfying, in the sense that it was not possible to obtain an ARX model whose response fitted the data collected in more than 50%, no matter the orders used. So as to accomplish a better fit, a change of the input variable is considered: instead of using the gate position, which entails nonlinearities, the flow under the gate is used, which relates to the position, $p_i(t)$, of gate $i$ by

$$q_i(t) = C_{ds}Wp_i(t)\sqrt{2g(h_{upstream,i} - h_{downstream,i})},$$

(4)

where $q_i$ is the flow under gate $i$, $C_{ds}$ is the discharge coefficient, $W = 0.49$ m is the width of the gates, $h_{upstream,i}$ is the water level upstream of gate $i$ and $h_{downstream,i}$ is the water level downstream of gate $i$ [3]. The value of the discharge coefficient is incorporated in the static gain of the system.

Figure 9 shows the flow under the gates, obtained with equation (4) from the gate positions represented in figure 8.

In fact, after this change of variables, the results obtained are much more satisfying, since the ARX model response fits the data collected from the nonlinear model in more than 95%, as shown in figure 10. For this reason, henceforward identification of MIMO systems is to be performed using the flow under the gate instead of its position.

Moreover, the results obtained also prove that the matrix structures as-
Figure 8: Position of gates 1, 2 and 3 in the open-loop simulation of the canal nonlinear model. A PRBS is applied to each gate position around the equilibrium values 5.96 cm, 6.01 cm and 6.09 cm, for gates 1, 2 and 3, respectively, while the fourth gate is kept at the position 28.77 cm.

sumed for $A(q^{-1})$ and $B(q^{-1})$ are acceptable. The orders and delays of the ARX model whose response is presented in figure 10 are:

$$N_A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}, \quad N_B = \begin{bmatrix} 4 & 3 & 0 \\ 3 & 4 & 4 \\ 0 & 3 & 3 \end{bmatrix}, \quad N_K = \begin{bmatrix} 1 & 2 & 0 \\ 1 & 1 & 2 \\ 0 & 1 & 1 \end{bmatrix}.$$

After converting the ARX model obtained to a state-space model such that

$$x(k+1) = Ax(k) + Bu(k)$$

$$y(k) = Cx(k),$$

matrices $A$, $B$ and $C$ will have the following structure

$$A = \begin{bmatrix} A_{11} & 0 & 0 \\ 0 & A_{22} & 0 \\ 0 & 0 & A_{33} \end{bmatrix}, \quad B = \begin{bmatrix} B_{11} & B_{12} & 0 \\ B_{21} & B_{22} & B_{23} \\ 0 & B_{32} & B_{33} \end{bmatrix}, \quad C = \begin{bmatrix} C_{11} & 0 & 0 \\ 0 & C_{22} & 0 \\ 0 & 0 & C_{33} \end{bmatrix}.$$
Figure 9: Flow under gates 1, 2 and 3 in the open-loop simulation of the canal nonlinear model. A PRBS is applied to each gate position around the equilibrium values 5.96 cm, 6.01 cm and 6.09 cm, for gates 1, 2 and 3, respectively, while the fourth gate is kept at the position 28.77 cm.

4 LQG Control

The LQG controller consists in the combination of a Kalman filter, i.e., a linear-quadratic estimator (LQE) with a linear-quadratic regulator (LQR). The separation principle ensures that both can be designed and computed independently.

4.1 Linear-Quadratic Regulator

Consider a linear time invariant plant described by the state-space representation

\[ x(k + 1) = Ax(k) + Bu(k) \]

and output equation

\[ y(k) = Cx(k), \]

where \( k \) is a nonnegative integer that represents discrete-time, \( x \in \mathbb{R}^n \) is the state, \( u \in \mathbb{R}^m \) is the input variable, \( y \in \mathbb{R}^p \) is the output of the process and \( A \in \mathbb{R}^{n \times n}, B \in \mathbb{R}^{n \times m}, C \in \mathbb{R}^{p \times n} \) are matrices that define the parametrization of the model.

The linear-quadratic regulator problem consists in calculating the optimal gain matrix \( K \) such that the state-feedback law \( u(k) = -Kx(k) \) mini-
Figure 10: Comparison between the data collected from the open-loop simulation of the nonlinear model and the response of the ARX model, after filtering the data.
minimizes the cost function

\[ J = \frac{1}{2} \sum_{k=1}^{\infty} \left[ x^\top(k)Qx(k) + u^\top(k)Ru(k) \right], \tag{7} \]

where \( Q \in \mathbb{R}^{n \times n} \) is chosen such that \( Q = C^\top C \) and \( R \in \mathbb{R}^{m \times m} \) is a positive definite matrix, assumed to be diagonal.

The optimal state feedback gain \( K \) is

\[ K = (R + B^\top SB)^{-1} B^\top SA, \tag{8} \]

where \( S \) is the solution of the algebraic Riccati equation

\[ A^\top SA - S - A^\top SB(B^\top SB + R)^{-1} B^\top SA + Q = 0. \tag{9} \]

In order to assure that the system response follows a reference, integral action must be taken into account in the controller design. Figure 11 shows the schematic of the controlled system including integral action. \( T_s \) is the sample time.

![Schematic of a Linear-Quadratic Regulator including integral action.](image)

The plant in figure 11 is described by the dynamics in (5) and (6). From figure 11 we can also write

\[ x_I(k) = \frac{T_s}{z - 1} e(k) \Leftrightarrow x_I(k + 1) = x_I(k) + T_s e(k), \]

where \( e(k) = r - y(k) = r - Cx(k) \). Since we are designing a regulator, we consider \( r = 0 \) and the new state-space dynamics of the regulator can be written in matrix form as

\[ \begin{bmatrix} x(k + 1) \\ x_I(k + 1) \end{bmatrix} = \begin{bmatrix} A & 0 \\ -T_s C & 1 \end{bmatrix} \begin{bmatrix} x(k) \\ x_I(k) \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u(k) \tag{10} \]

with output

\[ u(k) = - \begin{bmatrix} k_x & k_I \end{bmatrix} \begin{bmatrix} x(k) \\ x_I(k) \end{bmatrix}. \tag{11} \]
4.2 Kalman Filter

Most of the times, however, it is not possible to have access to the state. Therefore, there is a need to estimate it, so that the control law mentioned in section 4.1 can be applied.

The estimator to be used is the Kalman filter, which is an optimal linear estimator, in the sense that it optimizes the signal/noise relation of the model. This is accomplished by finding an optimal gain matrix \( L \) that places the estimator poles conveniently.

Consider the discrete plant

\[
 x(k+1) = Ax(k) + Bu(k) + Gw(k)
\]

with measurements

\[
 y(k) = Cx(k) + v(k),
\]

where the process noise \( w(k) \) and measurement noise \( v(k) \) are random white noise sequences with zero mean, i.e.

\[
 E[w(k)] = E[v(k)] = 0,
\]

have no time correlation, that is

\[
 E[w(i)w^\top(j)] = E[v(i)v^\top(j)] = 0, \quad i \neq j,
\]

and have covariance matrices defined by

\[
 Q_n = E[w(k)w^\top(k)], \quad R_n = E[v(k)v^\top(k)].
\]

The Kalman filter estimates the state based on the state equation

\[
 \hat{x}(k+1) = A\hat{x}(k) + Bu(k) + L(y(k) - C\hat{x}(k)), \quad (12)
\]

where \( L \) is the optimal gain matrix given by

\[
 L = APC^\top(CPC^\top + R)^{-1},
\]

in that \( P \) is the solution of the Riccati equation

\[
 A^\top PA - P - A^\top PB(B^\top PB + R)^{-1}B^\top PA + Q = 0. \quad (13)
\]

5 Centralized Control

5.1 SISO Controller

Assuming full access to the state, in order to design a LQR, i.e. in order to find the optimal gain matrix \( K \), mentioned in section 4.1, the parameter \( R \)
needs to be defined. In the SISO case, this parameter is a positive integer, while in the multivariable case it is a positive definite matrix.

Figure 12 shows the effect of the variation of the parameter $R$ on the linear model response for the SISO case. Only pool 1 is taken into account. It is clear from the figure that the higher the value of this parameter, the slower the model responds. The value sought is the one that allows the model to respond as fast as possible, without leading to oscillations. Therefore, the value chosen for this case is $R = 1000$.

![Effect of the variation of $R$ on the linear model response](image)

Figure 12: Closed-loop response of the controlled linear SISO model that takes into account the water level of pool 1 and gate 1, for different values of $R$, considering full access to the state. Gates 1, 2 and 3 are kept at the positions 6.01 cm, 6.09 cm and 28.77 cm, respectively.

In figure 13 the same simulation but for the nonlinear model is shown. The difference verified is mainly that in the nonlinear model there are more oscillations, which is particularly evident for $R = 100$. Also from this figure, we can see that from the three values of $R$, $R = 1000$ is the best choice.

Once the LQR is designed, the estimator becomes the main focus. As seen in section 4.2, the Kalman filter makes use of two covariance matrices that need to be determined: $R_n$ and $Q_n$. The first has the same influence on the response as $R$ in the LQR design, while the latter can be approximated by $q^2BB^\top$, where $q$ is a positive integer. Since we are considering only the SISO case, $R_n$ is a positive integer as well. To study the influence of each of these parameters in both the linear and the nonlinear models response, the controlled models are simulated, keeping $R = 1000$. First, the value of $q$ is
Effect of the variation of $R$ on the nonlinear model response

Figure 13: Closed-loop response of the controlled nonlinear SISO model that takes into account the water level of pool 1 and gate 1, for different values of $R$, considering full access to the state. Gates 1, 2 and 3 are kept at the positions 6.01 cm, 6.09 cm and 28.77 cm, respectively.

fixed at $q = 1$ and the influence of $R_n$ is studied. Figure 14 shows the result of the controlled linear SISO model simulation, in these conditions.

From figure 14 we can see that the lower the value of $R_n$, the higher the overshoot. Similar to what happened with $R$ in the LQR case, $R_n$ determines the speed of the estimator, which means that the lower the value of $R_n$, the faster the estimator and the higher the overshoot verified in the model response. However, the difference between the curves obtained for $R_n = 10$ and $R_n = 100$ is much less significant when compared to the difference obtained with $R_n = 1$ and $R_n = 10$. In fact, for higher values of $R_n$, the difference is barely detectable. Therefore, the value chosen is $R_n = 100$.

In figure 15, the same simulation but for the nonlinear model is presented. Once again, the only difference for the linear case is the more pronounced oscillation. The figure also shows that these oscillations decrease as the value of $R_n$ becomes higher, which reinforces the choice made.

To study the influence of parameter $q$ in both models responses, the value of $R_n$ is now kept constant at the value previously chosen, $R_n = 100$. The simulation of the linear model is shown in figure 16. From this figure, specially from the time evolution of the gate position, we can see that increasing the value of $q$ leads to oscillatory behavior, which is once again much more pronounced for the response of the nonlinear model, shown in
Figure 14: Closed-loop response of the controlled linear SISO model that takes into account the water level of pool 1 and gate 1, with estimated state, for different values of $R_n$, while keeping $R = 1000$ and $q = 1$. Gates 1, 2 and 3 are kept at the positions 6.01 cm, 6.09 cm and 28.77 cm, respectively.

From both figures, we can also see that for values of $q$ higher than 100, no significant differences are detected. In fact, from the frequency response of the linear model, shown in figure 18, it is clear that as the value of $q$ increases the bandwidth becomes larger, which results in a faster response, and that the responses for $q = 100$ and $q = 1000$ overlap. The value of $q$ chosen must be the highest value that does not lead to oscillatory behavior. Therefore, the selected value is $q = 1$.

5.1.1 Constraining closed-loop poles

In order to try to overcome the problems of overshoot and oscillatory behaviors seen, it is also possible to constrain the poles to a circle of radius lower than 1. This can be accomplished by minimizing the cost function

$$J = \frac{1}{2} \sum_{k=1}^{\infty} \left[ x^T(k)Qx(k) + u^T(k)Ru(k) \right] \alpha^{2k},$$

(14)

in that $1/\alpha$, $\alpha > 1$, is the new circle radius. The corresponding state-space dynamics is now

$$x(k+1) = \alpha Ax(k) + \alpha Bu(k),$$

(15)

as shown in [2].
Figure 15: Closed-loop response of the controlled nonlinear SISO model that takes into account the water level of pool 1 and gate 1, with estimated state, for different values of $R_n$, while keeping $R = 1000$ and $q = 1$. Gates 1, 2 and 3 are kept at the positions 6.01 cm, 6.09 cm and 28.77 cm, respectively.

Figures 19 and 20 show the influence of $\alpha$ on the linear and the nonlinear models responses, respectively. As observed, the models responses are very sensitive to and largely affected by this parameter. In fact, for values above 1.005, the model response no longer converges to the reference value. It is also noticeable from those figures that, when $\alpha = 1.007$, the response is closer to the others in the nonlinear model case, which reflects the robustness of the projected LQG controller. The value of $\alpha$ to be used is $\alpha = 1.005$, since that is the highest value that allows the response to follow a reference.

Figures 21 and 22 show the controlled linear and nonlinear models time responses, respectively, when the controller parameters assume the previously computed values: $R = 1000$, $R_n = 100$, $q = 1$ and $\alpha = 1.005$.

5.2 MIMO Controller

In centralized control, only one controller is used. In the present case, the controller to be designed has three inputs, which are the flows under each gate considered (1, 2 and 3), and three outputs, which are the water levels of pools 1, 2 and 3.

The MIMO LQG controller design follows the same procedure as for the SISO case, with the difference that in the MIMO case $R$ and $R_n$ become
Effect of the variation of $q$ on the linear model response

Figure 16: Closed-loop response of the controlled linear SISO model that takes into account the water level of pool 1 and gate 1, with estimated state, for different values of $q$, while keeping $R = 1000$ and $R_n = 100$. Gates 1, 2 and 3 are kept at the positions 6.01 cm, 6.09 cm and 28.77 cm, respectively.

Matrices $n_y \times n_y$, in that $n_y$ is the number of outputs. As a matter of simplicity all entries of the diagonals of matrices $R$ and $R_n$ are assumed to be equal. Taking into account that three outputs are considered, that means

$$
R = \begin{bmatrix}
    r & 0 & 0 \\
    0 & r & 0 \\
    0 & 0 & r
\end{bmatrix}, \quad
R_n = \begin{bmatrix}
    r_n & 0 & 0 \\
    0 & r_n & 0 \\
    0 & 0 & r_n
\end{bmatrix}.
$$

The previously designed controller is applied to the MIMO nonlinear model identified in section 3.2. The parameters used are the same as in the SISO case, i.e. $r = 1000$, $r_n = 100$, $q = 1$ and $\alpha = 1.005$, since they lead to a similar response for the MIMO case as they did for the SISO case and they proved to be the best. The simulation results are shown in figure 23.

5.2.1 Offtakses

In order to test the ability of the designed controller to respond to the offtakes, that same controller is applied to the MIMO nonlinear model considered in section 5.2, while the offtake flows of pools 1, 2 and 3 change between 0 l/s and 1 l/s, at chosen time instants. Figures 24 and 25 show the results of this simulation.
Figure 17: Closed-loop response of the controlled nonlinear SISO model that takes into account the water level of pool 1 and gate 1, with estimated state, for different values of $q$, while keeping $R = 1000$ and $R_n = 100$. Gates 1, 2 and 3 are kept at the positions $6.01\text{ cm}$, $6.09\text{ cm}$ and $28.77\text{ cm}$, respectively.

From figure 25, we can see that the position of gate 1 is barely affected by the other pools, it only changes when the flow of offtake 1 varies. As expected, when the offtake flow increases, the gate closes, so as to drain a lower quantity of water to the next pool. On the other hand, if we observe the time evolution of the position of gate 2, we notice that the position changes whenever the flows of offtakes 1 and 2 vary, but that it is barely affected by offtake 3. At last, it is clear that the position of gate 3 changes whenever any of the offtakes is turned on.

It is also noticeable that a significant overshoot occurs in the position of gate 2 when the flow of offtake 2 decreases to $0\text{ l/s}$. This happens every time all three offtakes are draining water and one of them stops. In fact, we can see that when all offtakes are on gate 3 is fully closed and from figure 24 we see that within that period the water levels of all three pools are beneath the desired value. This means that, theoretically, in order to attain the desired water levels, gate 3 should be at a negative position. Therefore, when one of the offtakes is turned off, the water existent within the three pools becomes sufficient to satisfy the water levels requirements again and the gates over respond, especially the gate that corresponds to the offtake that has been turned off and the one(s) located downstream of it. That also explains the overshoots observed in the water levels evolutions shown in figure 24. For
Figure 18: Frequency response of the controlled linear SISO model that takes into account the water level of pool 1 and gate 1, with estimated state, for different values of $q$, while keeping $R = 1000$ and $R_n = 100$. Gates 1, 2 and 3 are kept at the positions 6.01 cm, 6.09 cm and 28.77 cm, respectively.

this reason, we can conclude that offtake flows of 1 l/s are too high if the purpose is that all three pools can be serving at the same time. Apart from that, the water levels do not suffer significant changes due to the offtakes.

6 Control system tolerant to actuator faults

As previously stated, we assume that only gate 2 is susceptible of failing, that is getting stuck at a fixed position independently of the instruction received. In order to design a control system tolerant to such a fault, two centralized LQG controllers are considered: the one designed in section 5.2 for the case that the system is operating regularly and another one for the case that the above fault is detected.

6.1 System identification assuming actuator faults

To begin with, similar to the procedure followed for the controller used for the system in regular operation, the nonlinear model is identified, keeping gate 2 at a fixed position. Only the water levels of pools 1 and 3 are taken into account. Similar to what was done in the identification process of the MIMO model with three inputs and three outputs in section 3.2, as a valid
Figure 19: Closed-loop response of the controlled linear SISO model that takes into account the water level of pool 1 and gate 1, with estimated state, for different values of $\alpha$, while keeping $R = 1000$, $R_a = 100$ and $q = 1$. Gates 1, 2 and 3 are kept at the positions 6.01 cm, 6.09 cm and 28.77 cm, respectively.
Figure 20: Closed-loop response of the controlled nonlinear SISO model that takes into account the water level of pool 1 and gate 1, with estimated state, for different values of $\alpha$, while keeping $R = 1000$, $R_n = 100$ and $q = 1$. Gates 1, 2 and 3 are kept at the positions $6.01\, cm$, $6.09\, cm$ and $28.77\, cm$, respectively.
Figure 21: Closed-loop response of the controlled linear SISO model that takes into account the water level of pool 1 and gate 1. The LQG parameters are $R = 1000$, $R_n = 100$, $q = 1$ and $\alpha = 1.005$. Gates 1, 2 and 3 are kept at the positions $6.01 \text{ cm}$, $6.09 \text{ cm}$ and $28.77 \text{ cm}$, respectively.

Simplification the order matrices are assumed to be

$$N_A = \begin{bmatrix} n_{a_1} & 0 \\ 0 & n_{a_2} \end{bmatrix}, \quad N_B = \begin{bmatrix} n_{b_1} & n_{b_2} \\ n_{b_3} & n_{b_4} \end{bmatrix}, \quad N_K = \begin{bmatrix} n_{k_1} & n_{k_2} \\ n_{k_3} & n_{k_4} \end{bmatrix},$$

for the same reason as before: the water level of pool 1 is mostly affected by the value it assumes in the previous time instant than by that of pool 3, just like pool 3 is mostly affected by its own water level in the previous time instant.

Figures 26 and 27 show the time evolution of the water levels of pools 1 and 3 and the flow under the respective gate, when gate 2 is kept at a height of $6 \text{ cm}$ and gate 4 at $28.77 \text{ cm}$. The value of $6 \text{ cm}$ was selected as it is the equilibrium value around which this study is being carried out.

The nonlinear model with two inputs (position of gates 1 and 3) and two outputs (water levels of pools 1 and 3) is identified and the results obtained for the fit between the data and the linear model are shown in figure 28. The identification makes use of the least squares method as well and the orders that lead to the presented results are

$$N_A = \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix}, \quad N_B = \begin{bmatrix} 4 & 3 \\ 4 & 4 \end{bmatrix}, \quad N_K = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}.$$
Figure 22: Closed-loop response of the controlled nonlinear SISO model that takes into account the water level of pool 1 and gate 1. The LQG parameters are $R = 1000$, $R_n = 100$, $q = 1$ and $\alpha = 1.005$. Gates 1, 2 and 3 are kept at the positions 6.01 cm, 6.09 cm and 28.77 cm, respectively.

If we do not consider the approximation of the $N_A$ matrix being diagonal, the best fits we obtain do not improve significantly (under 2%). This result is shown in figure 29 and the order matrices that lead to it are

\[ N_A = \begin{bmatrix} 2 & 1 \\ 2 & 4 \end{bmatrix}, \quad N_B = \begin{bmatrix} 4 & 4 \\ 3 & 4 \end{bmatrix}, \quad N_K = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}. \]

6.2 LQG controller design assuming actuator faults

Once the system is identified, the LQG controller with the same parameters selected in section 5.2 is applied to the nonlinear model. In figure 30 the time evolution of water levels and gate positions is shown for the three pools, when gate 2 is stuck at a height of 6 cm.

From figure 30, we see that the water levels of pools 1 and 3 follow the respective reference, which does not occur in pool 2. Instead, the water level of the latter follows the behavior of the water level of pool 3, showing a slightly more pronounced overshoot at the changes of the position of gate 1. However, since gate 2 is stuck at the equilibrium level that leads to the set point of the considered reference, the values between which the water level of pool 2 oscillate are those of the reference. If, on the other hand, gate 2 stops moving at a different position, for instance at a height of 10 cm, the water
Figure 23: Closed-loop response of the controlled nonlinear MIMO model that takes into account the water level of pools 1, 2 and 3 and respective gates, while gate 4 is kept at the fixed position of 28.77 cm. The controller parameters are $r = 1000$, $r_n = 100$, $q = 1$ and $\alpha = 1.005$. 
Figure 24: Closed-loop response of the controlled nonlinear MIMO model that takes into account pools 1, 2 and 3, when the offtake flows associated to those pools change between 0 l/s and 1 l/s, at chosen time instants. Gate 4 is kept at the fixed position of 28.77 cm.
Figure 25: Positions of gates 1, 2 and 3, in the closed-loop simulation of the controlled nonlinear MIMO model that takes into account pools 1, 2 and 3, when the offtake flows associated to those pools change between 0 l/s and 1 l/s, at chosen time instants. Gate 4 is kept at the fixed position of 28.77 cm.
Figure 26: Open-loop response of the nonlinear MIMO model that takes into account pools 1 and 3, when gate 2 is kept at a height of 6 cm and gate 4 at 28.77 cm.

Figure 27: Flow under gates 1 and 3 in the open-loop simulation of the nonlinear MIMO model that takes into account pools 1 and 3, when gate 2 is kept at a height of 6 cm and gate 4 at 28.77 cm.
Figure 28: Comparison between the data collected from the open-loop simulation of the nonlinear model and the response of the ARX model. The inputs are the positions of gates 1 and 3, the outputs are the water levels of pools 1 and 3 and gate 2 is stuck at a height of 6 cm, while gate 4 is kept at a position of 28.77 cm.
Figure 29: Comparison between the data collected from the open-loop simulation of the nonlinear model and the response of the ARX model, without assuming simplifications in the order matrices. The inputs are the positions of gates 1 and 3, the outputs are the water levels of pools 1 and 3 and gate 2 is stuck at a height of 6 cm, while gate 4 is kept at a position of 28.77 cm.
Figure 30: Closed-loop response of the controlled nonlinear MIMO model that takes into account pools 1 and 3, when gate 2 is stuck at a height of 6 cm and gate 4 is kept at the fixed position of 28.77 cm. The controller parameters are $r = 1000$, $r_n = 100$, $q = 1$ and $\alpha = 1.005$. 
level of pool 2 decreases under its equilibrium value, however showing the same behavior. In this case, which is shown in figure 31, we can also notice that the water level of pool 3 presents a slight overshoot at the changes of the position of gate 1, similar to what happens to the water level of pool 2. In fact, comparing water levels of pools 2 and 3, we see that they are almost equal.

At last, if we open gate 2 completely, i.e. at the position of 80 cm, we see from figure 32 that the water level of pool 2 decreases under that of pool 3. In this case, pools 2 and 3 are considered as one and, due to the existing, however slight, bottom slope, the water is drained to pool 3, leading its level to rise. This is actually a limitation of the nonlinear model of the canal, since it does not really occur in reality, due to the water propagation delay inherent to its motion.
Figure 32: Closed-loop response of the controlled nonlinear MIMO model that takes into account pools 1 and 3, when gate 2 is stuck at a height of 80 cm and gate 4 is kept at the fixed position of 28.77 cm. The controller parameters are $r = 10000$, $r_n = 100$, $q = 1$ and $\alpha = 1.005$. 

34
6.3 Centralized multivariable control with actuator faults

The centralized control system consists of the two controller designed in sections 5.2 and 6.2, but also of an entity that controls the switch between the two controllers when a fault is detected, a supervisor. However, for now, we will simulate a fault in gate 2 just by choosing a time instant for it to occur.

Figure 33 shows the results of the simulation of the control system tolerant to faults in the actuator of gate 2, when the fault occurs at the time instant 5000 s. From the figure we see that the water levels of pools 1 and 3 follow the respective references, without major oscillations at the instant of the transition between controllers.

As to water level of pool 2, it follows the corresponding reference until the fault occurs. Afterwards, its behavior follows the one observed in figures 31 and 32, however presenting a higher equilibrium level. This is due to the fact that the position at which gate 2 stopped moving is $5.2 \text{ cm}$, which is lower than the equilibrium position ($6 \text{ cm}$).

One should notice that a decrease of $8 \text{ mm}$ in the gate position caused the water level to rise $5 \text{ cm}$. In fact, since we are dealing with very low gate heights, a decrease in the gate position leads to a much more significant rise in the water level of the corresponding pool. For positions of gate 2 under $5 \text{ cm}$, the water levels of pools 1 and 3 no longer follow the references and for even lower values (under $3.5 \text{ cm}$) the water levels of pools 1 and 2 overflow.

In the last simulation, water levels were supposed to follow a periodic square signal. However, usually what is intended is that a certain water level is maintained in each pool, even when water is being extracted from the canal. With that in mind, the nonlinear model is simulated subject to the centralized control system tolerant to faults in the actuator of gate 2, while offtake flows of pool 1, 2 and 3 vary from $0/\text{s}$ to $1/\text{s}$, at chosen time instants. The reference values for the water levels of pools 1, 2 and 3 are kept constant at the set points, i.e. at $70 \text{ cm}$, $60 \text{ cm}$ and $50 \text{ cm}$, respectively. Figures 34 and 35 show the results of that simulation.

As expected, levels 1 and 3 follow the reference without significant deviations, the same not happening to the level of pool 2. If we observe the time evolution of the position of gate 3 in figure 35, we notice that its behavior is the same as in the case that no fault occurs. In fact, the latter presents as many transitions between gate positions as there are offtake transitions and it only presents three different positions: one for the case that no offtakes are draining water; a second one for the case that only one of them is; and one last position for the case that two of the offtakes are draining water. These are the only possibilities in this simulation.

As to pool 1, a difference is noticed from what was seen in figure 25 in section 5.2.1. The conclusion we reached of pool 1 not being affected by
Figure 33: Closed-loop response of the controlled nonlinear MIMO model that takes into account pools 1, 2 and 3, considering the designed control system tolerant to faults in the actuator of gate 2. The fault occurs at the time instant 5000 s.
Figure 34: Closed-loop response of the controlled nonlinear MIMO model that takes into account pools 1, 2 and 3, considering the designed control system tolerant to faults in the actuator of gate 2. The fault occurs at the time instant 5000 s. The reference values for the water levels of pools 1, 2 and 3 are constant and equal to 70 cm, 60 cm and 50 cm, respectively. Offtakes of pools 1, 2 and 3 are switched on and off at chosen time instants.
Figure 35: Gate positions in the closed-loop simulation of the controlled nonlinear MIMO model that takes into account pools 1, 2 and 3, considering the designed control system tolerant to faults in the actuator of gate 2. The fault occurs at the time instant 5000 s. The reference values for the water levels of pools 1, 2 and 3 are constant and equal to 70 cm, 60 cm and 50 cm, respectively. Offtakes of pools 1, 2 and 3 are switched on and off at chosen time instants.
pool 2 no longer applies. In fact, from figure 35 we see that when offtake 2 is switched on, the position of gate 1 slightly decreases, and that when offtake 1 stops draining water the position of gate 1 does not return to its original value. That only occurs when offtake 2 is off again.

If we now compare both figures, we notice that, when the fault occurs, at instant 5000 s, water level of pool 2 continues following the reference to decrease when offtake 1 is turned on. When offtake 3 is afterwards turned off, water level of pool 2 does not change significantly, which means that pool 2 is still barely affected by pool 3. Instead, water level 2 only reacts to changes in offtakes 1 and 2: when offtake 2 turns on, it decreases, to rise again when offtake 1 is off and it finally returns to its original value when offtake 2 is switched off.

7 Experimental results

In this section some experimental results are presented.

Figure 36 shows the closed-loop response of the canal when it is controlled with a centralized multivariable LQG controller. To begin with, the controller parameter $r$ was set to $10^4$.

From figure 36 we see that the water levels follow the respective references, only with a slight oscillation. It is also evident the coordination between the gates. For example, around time instant $0.36 \times 10^4$ s, when the reference of water level 2 rises, gate 2 closes as to retain the water, but also gate 1 opens in order to draw a higher quantity of water to pool 2. Or, another example, when the reference of water level 3 decreases around time instant $1.08 \times 10^4$ s, gate 3 opens to draw more water at the same time that gate 2 closes to maintain the water level of pool 2.

From a more global point of view, if we take attention to the time evolution of the position of gate 1, we notice that it is mostly influenced by the changes in the references of pools 1 and 2 and barely by the reference of pool 3. On the other hand, the time evolution of the position of gate 2 shows that gate 2 changes its position every time any of the reference changes. As to gate 3, it changes its position mostly when a change in the reference of pool 3 occurs, but also when reference 2 varies. This is in accordance to the assumptions made for the identification is sections 3.2 and 6.1.

Figure 37 shows the closed-loop response of the canal controlled with a centralized multivariable LQG controller, until a fault in gate 2 occurs (red mark). The occurrence of a fault consists in the blockade of gate 2 at a chosen time instant. When a fault occurs, the position of gate 2 remains constant regardless of the command received by the controller. After the fault is detected (yellow mark), the control system is reconfigured so that only the water levels of pools 1 and 3 are controlled. This is accomplished by switching to a second centralized multivariable LQG controller, designed
Figure 36: Closed-loop response of the canal subject to a centralized multi-variable LQG controller, considering only pools 1, 2 and 3. The controller parameters are $r = 1000$, $r_n = 1000$, $q = 1$ and $\alpha = 1.004$. Water levels above, gate positions below.
as explained in section 6, which is responsible for keeping the water levels of pools 1 and 3 at the desired values.

The parameter $r$ of the first controller (before fault occurrence) was set to $10^3$, so as to make the system response faster in relation to that of figure 36. The parameter $r$ of the second controller (after fault occurrence) was also set to $10^3$.

From figure 37 we see that the water levels of pools 1 and 3 follow the respective references, again only with a slight oscillation. After the red and yellow marks, when the second controller is the one in operation, the coordination between gates 1 and 3 is evident. Around time instant 5400s, when the reference value of pool 1 decreases, gate 1 opens so as to draw more water from pool 1 (which makes water level of pool 2 to rise), at the same time that gate 3 opens, so that the water level of pool 3 is maintained.

In figure 37 we also see that the fault detection happens only a few seconds after the occurrence of the fault. Since in real experiments the command sent to the gate and the actual gate position are not equal most of the time, the fault detection criterion cannot be based on the fact that the difference between those two values has to be zero. In order to detect the occurrence of a fault, the following two step algorithm was developed: First, for each gate $i$, $i = 1, 2, 3$, an error $\tilde{u}_i$ between the command sent to the gate, $u_i$, and the actual gate position, $u_{r,i}$, must be defined,

$$\tilde{u}_i(k) = u_i(k) - u_{r,i}(k).$$  \hfill (16)

After that, a performance index, $\Pi$, can be obtained from this error by

$$\Pi(k) = \gamma \Pi(k-1) + (1 - \gamma) |\tilde{u}_i(k)|$$  \hfill (17)

A detection of a fault happens whenever $\Pi(k) \geq \Pi_{max}$, where $\Pi_{max}$ is a chosen threshold.

Figure 38 shows the time evolution of $\Pi$ and the threshold (above) and a fault indicator, $I_F$ (below). The fault indicator equals one if a fault has been detected and equals zero otherwise. In the present experiment, the values of $\Pi_{max}$ and $\gamma$ chosen were $\Pi_{max} = 5$ and $\gamma = 0.95$ and the fault was detected after approximately two minutes.

The values of $\gamma$ and $\Pi_{max}$ chosen must be such that the fault is detected as fast as possible and without the occurrence of false alarms, i.e., when a fault is detected but no fault has occurred. Figure 39 shows the effect of the variation of $\gamma$ and $\Pi_{max}$ on the number of false alarms and on the time elapsed since a fault occurred until it was detected (hereafter denoted as detection time), for the experiment of figure 37. Plots (a) and (b) show the number of false alarms and the detection time as a function of $\gamma$, respectively, while keeping $\Pi_{max} = 5$. In plot (c) we see the number of false alarms as a function of the detection time, when $\gamma$ varies between 0 and 1, while
Figure 37: Closed-loop response of the canal subject to a centralized multivariable LQG controller. Reconfiguration after a fault in gate 2 is detected (yellow mark). Both controllers parameters are $r = 1000$, $r_n = 100$, $q = 1$ and $\alpha = 1.004$. Water levels above, gate positions below.
\(\Pi_{\text{max}} = 5\). When, instead of varying \(\gamma\) we vary the threshold \(\Pi_{\text{max}}\), while keeping \(\gamma = 0.95\), we obtain plots (d), (e) and (f).

From plot (a) we conclude that the higher the value of \(\gamma\) the fewer the number of false alarms we obtain. The first value of \(\gamma\) that ensures that no false alarms occur for this experiment is \(\gamma = 0.913\). Now switching our attention to plot (b), we notice that the detection time decreases until \(\gamma = 0.913\), after what it starts increasing slowly until \(\gamma \approx 0.96\) and afterwards it increases sharply towards infinity. In fact, when \(\gamma = 1\), the performance index \(\Pi\) equals zero, which means that the fault is never detected and that no false alarms occur. From plot (c) we see that the point with the lowest detection time and the lowest number of false alarms corresponds to \(\gamma = 0.913\), which would then be the best choice for this experiment. In order to assure that no false alarms occur for other experiments we need to choose a higher value of \(\gamma\), taking into account that it should be lower than 0.96, so that the fault is detected in a reasonable period of time. Therefore, the value chosen was \(\gamma = 0.95\).

Following the same reasoning, from plot (d) we notice that the minimum threshold that ensures that no false alarms occur for this experiment is 4.58. As to the detection time, in plot (e) we see that it is minimum for that same value of \(\gamma\), after what is increases slightly until \(\Pi_{\text{max}} \approx 7\) and afterwards it increases sharply from less than 2\(\text{min}\) to 47\(\text{min}\). In plot (f) the value of the threshold that corresponds to the lower detection time and no false alarms is
The chosen value, $\Pi_{\text{max}} = 5$, assures that no false alarms occur, while guaranteeing that the fault is detected in a reasonably short period of time.

![Graphs](a) - (f)

Figure 39: Effects of the variation of $\gamma$ on the number of false alarms and the time for detection, while keeping $\Pi_{\text{max}} = 5$ (a, b, c), and effects of the variation of $\Pi_{\text{max}}$ on the number of false alarms and the time for detection, while keeping $\gamma = 0.95$ (d, e, f).

8 Conclusions

The nonlinear model of the NuHCC facilities canal was identified using an ARX structure, first considering only pool 1 and afterwards pool 1, 2 and 3, so that LQG controllers could be applied to the resulting linear models. The SISO identification of pool 1 and corresponding gate was useful to study the effect of the variation of the LQG parameters $R_s$, $R_m$ and $q$ and also parameter $\alpha$ on the model response. The MIMO identification was used to design a three input/three output LQG controller, that was able to follow references without significant oscillations. It was also seen that the designed LQG managed to control the three inputs/three outputs system when oifftakes are turned on and off, as long as the quantity of water being extracted from the canal did not cause any of the gates to fully close. From what we observed, this meant that the three oifftakes could not be draining a flow of water of 1 l/s at the same time.

In order to deal with faults in gate 2, the nonlinear model of the canal
was identified, considering the positions of gates 1 and 3 as inputs and the respective water levels as outputs, while gate 2 was stuck at a fixed position. The LQG controller previously designed was applied to the resulting linear model and it was able to control the water levels of pools 1 and 3, but not of pool 2. It was seen that the water level of pool 2 followed the behavior of that of pool 3, assuming the same values as the latter when gate 2 was fixed at the position of 10 cm. For higher values of the position of gate 2, the water level of pool 2 decreased under that of pool 3 and, for lower positions, it increased until it overflowed, for positions under 3.5 cm. As to the other pools, their water levels followed the references as long as the position of gate 2 was not under 5 cm.

In the simulation of the control system composed of both controllers, when a fault occurred, the water levels of pools 1 and 3 still followed the respective references, without significant oscillations in the transition. As to water level of pool 2, it followed the reference, as expected, until the fault occurred and, afterwards, adopted the behavior of the water level of pool 3, however at a higher equilibrium level. This was also accomplished through a smooth transition.

When, instead of applying a periodic reference to the system input, the controlled system tolerant to faults in gate 2 was simulated while the offtake flows of pools 1, 2 and 3 varied, pools 1 and 3 followed the references, but not pool 2. Opposite to what was seen in the regular operation case when it was tested to changes in offtake flows, the position of gate 1 was now affected by the offtake of pool 2. On the other hand, the evolution of the position of gate 3 followed the same behavior as in the regular operation case, not denunciating the occurrence of a fault.

Experiments on the canal showed that the water levels of all pools followed the respective references in the regular operation case. Still relating to this experiment, the coordination between the gates was evident from the plots presented and the interaction observed between the gate positions and the water levels confirmed the assumptions made for the identification: the water levels are independent from each other and the position of gate 1 does not affect the water level of pool 3, the same way as the position of gate 3 does not affect the water level of pool 1. As to the case where a fault occurred, as expected, only water levels of pools 1 and 3 followed the respective references. In this case the coordination between gates 1 and 3 was evident.

The detection of a fault was accomplished through a performance index based on the difference between the command sent to a gate and its real position. When this performance index exceeded a certain threshold, a fault was detected. It was seen that both the threshold and the performance index had to be chosen such that a fault was detected as soon as possible, while ensuring that no false alarms occurred.
References


