Fractional signal processing and applications

The term “fractional” emerged in recent years in connection with different signal processing theories and techniques, sometimes with no visible connection between them. Nevertheless, the first reference to this area appeared during 1695 in a letter from Bernoulli to Leibnitz, where he formulated a question about the meaning of a non-integer order derivative. It was the beginning genesis of the Fractional Calculus that is the root of the continuous-time fractional systems described by fractional differential equations. Since then, Fractional Calculus (FC) evolved through the contributions of many famous mathematicians. In spite of the progress in pure mathematics, we had to wait up to the 1920s, during the 20th century, for the appearance of studies concerning the use of FC in Applied Sciences. Furthermore, only in the last three decades the application of FC engineering deserved attention, motivated by the works of Mandelbrot on Fractals, that led to a significant impact in several scientific areas and attracted, definitively, the attention to fractional “objects”. Presently, new themes are the object of active research such as, fractional Brownian motion, discrete-time fractional linear systems, fractional delay filtering, fractional splines and wavelets.

In a similar line of thought, we can mention the concept of fractional fourier transform (FF) that was first introduced in 1929 in the area of mathematics. More recently, in 1980, this tool was rediscovered by the physicist Namias based on the spectral structure of the classical fourier transform. Most reported applications on FF are in the field of optics but other topics in signal analysis have appeared in the literature, namely filtering, encoding, watermarking, phase retrieval and others. While FC and FF constitute distinct mathematical formalisms some results point out that a common paradigm can be achieved, although not totally clear at the present state of affairs.

Bearing these facts in mind, we felt that it was the time for establishing a special issue on the application of FC and FF concepts in areas related to signal processing. We believe that the contributions gathered in this issue give, not only a good overview on the state of application of FC and FF, but also point out new directions of future research and development. Consequently, in order to guide the reader throughout the issue we have grouped the articles in five major areas, leading from theoretical aspects up to more applied studies as listed in the sequel.

Theoretical achievements in fractional calculus

T.T. Hartley and C.F. Lorenzo introduce the concept of continuous-order distribution and discuss its application in the identification of fractional and integer order systems.

M. Ortigueira studies the initial condition problem for fractional linear system and extends the well-known result obtained by the Laplace Transform, being the most interesting feature its independence upon the derivative definition. In a second paper, the same author proposes a new definition for a symmetric fractional B-spline that constitutes the generalization of the usual integer order B-spline.

Continuous-time realization and approximation

T. Poinot and J.-C. Trigeassou describe a method for the modelling and simulation of fractional systems, by adopting a state-space representation where the conventional integration is replaced by a fractional one.

N. Guijarro and G. Dauphin-Tanguy present a method for the approximation, reduction and realization of a class of fractional models. The method
is based on the interconnection of passive elementary blocks, leading to a finite-dimensional passive approximate model that can be reduced through a Krylov–Lanczos process. Finally, a bond graph realization of this reduced order model is given.

N. Heymans addresses hierarchical viscoelastic elements whose behaviour is intermediate between linear elasticity and Newtonian viscosity (spring pots). Such elements are incorporated into classical analogue models describing linear viscoelastic behaviour and the approach is extended to characterize the terminal transition from self-similar viscoelasticity up to pure flow.

Discrete-time realization and approximation

The very important topic of discrete-time realization and approximation of fractional systems is considered in the papers of Y. Chen and B.M. Vinagre and P. Ostalczyk, respectively. In the first article a discrete-time fractional differentiator is proposed by using a new family of first-order differentiators expressed in the pole-zero form. In the second study different types of discrete-time integrators and distinct orders of integration are considered. Their transient and frequency characteristics are also discussed.

Applications of fractional linear systems

Robotics: E.S. Pires, J.T. Machado, and P.M. Oliveira address the problem of signal propagation and fractional-order dynamics during the evolution of a genetic algorithm (GA) for generating robot manipulator trajectories.

P. Melchior, B. Orsoni, A. Poty, O. Lavialle, and A. Oustaloup present a comparison between two optimization methods for mobile robot path planning adopting a fractional potential. A fractional-order map of danger is embedded into A* and Fast Marching techniques for a vehicle path planning in an environment with fixed obstacles.

Diffusion: J.R. Leith studies the fractal growth of fractional diffusion from the viewpoint of the fractional derivative order influence on the scaling exponents. R. Gorenflo and A. Vivoli present another perspective and develop a theory of discrete-space discrete-time random walks, analogous to the theory of continuous-time random walks for space-time fractional diffusion equations.

Other: B. Mathieu, P. Melchior, A. Oustaloup, and Ch. Ceyral apply the fractional derivative in Image Processing by presenting a method for edge detection. On the other hand, R.R. Nigmatullin and S.I. Osokin present a complex permittivity model for dielectric asymmetric spectra description in dielectric spectroscopy signal processing.

Fractional fourier transform and applications

C. Candan and H.M. Ozaktas present new derivations of some sampling relations and series expansions for FF and other transforms. The method can also be applied to the Fresnel, Hartley, scale transform, and similar transforms.

L. Stankovic, T. Alieva, and M.J. Bastiaans propose a Wigner relative time-frequency distribution based on a fractional-Fourier-domain realization. This approach has the advantage of having reduced cross-terms and is generalized to the time-frequency distributions from the Cohen class.

G. Gonon, O. Richoux, and Claude Depollier use the FF transform to estimate the parameters of linear chirps. The authors adopt a filtering in the fractional domain and are successful in extracting linear chirps out of a multi-component noisy signal. The method is used to analyse the propagation of acoustic waves in a dispersive medium.