A QUADRATIC COST CONTROLLER TUNER BASED ON
PREDICTIVE ADAPTIVE CONTROL

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Abstract: The problem of controlling plants whose dynamics change sparsely in time, separated by long periods in which they are constant is considered. For that sake, an algorithm for controller tuning is proposed comprising two main blocks. One block performs the detection of plant dynamic changes while the other is a tuner based on the MUSMAR predictive adaptive control algorithm which optimizes quadratic costs. The controller gains update performed by MUSMAR is characterized by resorting to arguments which are an alternative to existing proofs. The tuner is exemplified in a simulated case study on position control of two coupled carts. © Controlo 2004.

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1. INTRODUCTION.

One aspect which hampers a wide application of adaptive control in industry is the fact that time varying gains, resulting from adaptation, may yield complicated dynamics. Indeed, an adaptive closed-loop system is a complex non-linear system for which some variables may slide along unknown manifolds. While this may be irrelevant in a simulation, the practical limitations of industrial processes may cause serious troubles when time varying controller gains exceed certain boundaries. Take as example the MUSMAR predictive adaptive controller (Greco et al., 1984), (Mosca, 1995). This algorithm has been applied with great success to widely different processes such as distributed collector solar fields (Coito et al., 1997), arc welding trailing centerline temperature (Santos et al., 2000) and superheated steam temperature control in a thermoelectric power plant unit (Silva et al., 2000). Both theory (Mosca et al., 1989) and a rich experience, in simulation and with power plants, support these applications and provide guidelines for selecting the parameters configuring the controller. It has been proven (Mosca et al., 1989) that the only possible convergence points are the local minima of the steady state quadratic cost constrained to the chosen controller structure. However, have not been established conditions under which the controller gains converge.

Since the controller structure is a priori chosen, the combination of model parameters which yields the same controller gains may not be unique and this may in turn cause the gains to slide. Furthermore, due to its multipredictor feature, there are much more parameters than controller gains. Although these identifiability problems can be solved up to some extend using directional forgetting (Kulhavý, 1987) there can be hazardous effects when some parameters cross zero.

The above problems are potentiated when the plant is operated for long periods of time (days, or even weeks as it happens in some industrial plants) at the same op-
erating point, since in this case the level of excitation for parameter identification can be exceedingly small.

On the other way, adaptation provides an essential tool for tuning when the complexity of the plant requires an high number of gains. To give one example, it is shown in (Silva et al., 2000) that an adaptive controller greatly outperforms a classical (optimized) controller whose parameters are manually tuned.

The above arguments motivate the consideration of a tuning structure consisting of two blocks. One block detects the changes in plant dynamics, which are assumed to occur sparsely in time and separated by periods in which it is constant. The other block is a tuner which, on the basis of plant data, selects the controller gains optimizing a given quadratic cost. Once the detector decides that a change in plant dynamics has occurred, the tuner starts adaptation in order to find the new optimal gains. When convergence is obtained which, on the basis of plant data, selects the controller whose parameters are manually tuned.

This algorithm is expected to "unite the best of two worlds". It provides adaptation, thereby allowing to control efficiently (slowly) time-varying plants. On the other side it also provides the safety of using constant gain controllers during long periods of operation.

The contribution of this paper consists in the presentation of such an algorithm and the exemplification of its advantages in a simulated case study related to position control of two coupled charts.

Furthermore, the gain update performed by the tuner considered is characterized using an alternative method with respect to published results, based on the ODE method.

The paper is organized as follows: after the introduction (this section) the predictive adaptive controller used as tuner is described in section 2, and its gain update properties are presented in section 3. The change detection and controller gains freezing algorithms are presented in section 4. Section 5 presents the case study and, finally, section 6 draws conclusions.

2. THE MUSMAR PREDICTIVE ADAPTIVE CONTROLLER.

Consider the problem of minimizing the steady-state quadratic cost

\[ J_\infty = \lim_{t \to \infty} E[y^2 + \rho u^2] \]  

where \( u \) and \( y \) are, respectively, the input and output of an ARMAX plant. In the realm of Predictive Adaptive Control, (1) is approximated by the multi-step receding horizon quadratic cost

\[ J_T(t) = E \left[ \sum_{k=1}^{T} y^2(t+k) + \rho u^2(t+k-1) \right] \]  

where \( T \geq 1 \) is an integer and \( E[\cdot|I^t] \) denotes the mean conditioned on the \( \sigma \)-field generated by the data \( I^t \) available up to discrete time \( t \).

The MUSMAR algorithm (Greco et al., 1984) aims at minimizing (2) and reads as follows:

1. Sample the plant output \( y(t) \) and compute the tracking error \( \hat{y}(t) \) with respect to the desired set-point \( y^*(t) \), by:

\[ \hat{y}(t) = y(t) - y^*(t) \]  

2. Using Recursive Least Squares (RLS), update the estimates of the parameters \( \theta_j, \psi_j, \mu_{j-1} \) and \( \phi_{j-1} \) in the following set of predictive models:

\[ \hat{y}(t+j) \approx \theta_j u(t) + \psi_j s(t) \]  
\[ u(t+j-1) \approx \mu_{j-1} u(t) + \phi_{j-1} s(t) \]  
\[ j = 1, \ldots, T \]  

where \( \approx \) denotes equality in least squares sense and \( s(t) \) is a sufficient statistic for computing the control, hereafter referred as the pseudo-state, given by

\[ s(t) = [\hat{y}(t) \ldots \hat{y}(t-n+1) u(t-1) \ldots u(t-m) w_1(t) \ldots w_1(t-nw_1) \ldots w_N(t) \ldots w_N(t-nw_N)]^T \]  

where the \( w_i \) are samples of auxiliary variables such as intermediate process variables or accessible disturbances. The choice of the variables and the number of their past samples entering \( s(t) \) defines the structure of the controller. The choice of \( n \) and \( m \) should be such that it allows to capture the dominant dynamics of the system. Too big values of \( n \) and \( m \) imply more parameters to estimate and this may lead to identifiability problems, in turn causing loss of control performance.

3. Apply to the plant the control given by

\[ u(t) = f^T s(t) + \eta(t) \]  

where \( \eta \) is a white dither noise of small amplitude and \( f \) is the vector of controller gains, computed from the estimates of the predictive models by

\[ F = -\frac{1}{\alpha} \left( \sum_{j=1}^{T} \theta_j \psi_j + \rho \sum_{j=1}^{T-1} \mu_j \phi_j \right) \]  

with the normalization factor \( \alpha \) given by

\[ \alpha = \sum_{j=1}^{T} \theta_j^2 + \rho (1 + \sum_{j=1}^{T-1} \mu_j^2) \]  

3. MINIMIZATION OF QUADRATIC COSTS.

The following proposition is proved in the appendix:

**Proposition 1**

Let \( F_0 \) be the control parameters applied since the remote past and let the vector of update parameters
Changes in plant dynamics are detected by taking the parameter $\epsilon$ The parameter $\epsilon$.

The controller gains is to improve the performance, since variable gains increase the transients, and to get a more uniform output variance, which is used to detect changes in the system parameters as it will be shown later. Another fact that also motivates the use of the freezer mechanism is the possibility of removing the dither which is no longer needed when adaptation stops. The gains are frozen, whenever the euclidean norm of the difference between the present gains, $F(t)$, and the mean of their past values divided by the mean, $F(t - 1)$, satisfies the inequality

$$\|F(t) - F(t - 1)\|_{F(t - 1)} \leq \epsilon$$  \hspace{1cm} (13)

The parameter $\epsilon$ should be chosen as a trade-off between a fast freezing (high $\epsilon$) and an accurate one (low $\epsilon$).

Changes in plant dynamics are detected by taking the plant output and comparing it with its mean plus or minus a constant times the output standard deviation (see Fig.1 to illustrate the mechanism), i.e.,

$$|y(t) - \bar{y}(t - 1)| \geq \pm k \sigma(t - 1)$$  \hspace{1cm} (14)

The constant $k$ should be chosen as a compromise between a fast detection and a false one. In the experiments, emphasis was placed in finding a value that reduces the false alarm probability to near zero.

The rationale behind this method is the following. After the gains freeze, in values that should stabilize the system, the output is bounded until a change in the system dynamics cause an increase in the output to values that go beyond the bounds.

After a change in plant dynamics is detected, the adaptation speed which results in an improvement of the transient response.

The mechanism performance can be improved by introduction of a waiting period $T_w$, after every change detection, to prevent large output oscillations (or even instability) originated by the covariance matrix resetting, and a fast “natural” freezing of the controller gains in values which are far from the “optimum”. This fast freezing is the outcome of large transients.

5. SIMULATION RESULTS.

In this section two experiments are carried out in order to compare the performances of MUSMAR with and without the change detection and controller gains freezing mechanism.

As a case study a benchmark example introduced in (Wie et al., 1992) is selected. Fig.2 represents this system made up by two masses connected by a spring, having an uncertain stiffness parameter $\gamma \in [0.23, 2]$. The control problem consists positioning the mass two on mass one, while the position of mass two $x_2(t)$, which is affected by an additive, bounded and uniformly distributed noise $w(t)$, is measured. This system can be represented in state-space form as:

\[ F = F_0 - \frac{1}{2\alpha(F_0)} R_s^{-1}(F_0)p(F_0) \]  \hspace{1cm} (10)

where

$$\alpha(F_0) \triangleq \sum_{i=1}^{T} \theta_i^2(F_0) + \rho(1 + \sum_{i=1}^{T-1} \mu_i^2(F_0))$$  \hspace{1cm} (11)

$$R_s(F_0) \triangleq E[s(t)s'(t)] = |\gamma|$$  \hspace{1cm} (12)

and $p(F_0)$ is the gradient of the receding horizon version of the cost (4), computed for $F = F_0$. The following proposition thus holds:

Proposition 2

For $T$ large enough, the only possible convergence points of MUSMAR are close approximations to the local minima of the steady state cost (1), given the controller structure imposed by the choice of the pseudo-state $s(t)$.

Proposition 2 provides the ground for the intuition according to which correct decisions will be made when an enlarged horizon is considered.
Hereafter, all numerical values are given in S.I. units.

In equations (15) \( x_i(t) \) and \( x_{i+1}(t) \) denote position and velocity, respectively of mass \( m_i \) \( (i = 1, 2) \). Hereafter, all numerical values are given in S.I. units.

It should be emphasized, that the continuous-time system must be sampled in order to apply the discrete-time MUSMAR algorithm. The sampling period was selected as 0.5s.

Henceforth, in order to test the algorithm capability to tackle sudden changes in the system dynamic behaviour, the continuous-time plant (15) is simulated with the stiffness parameter \( \gamma \) exhibiting stepwise time variations as represented in Fig. 3. The amplitude of the frequency characteristics of the continuous-time models for the four different stiffness values is shown in Fig. 4. The system is characterized by a vibration mode near unity subjected to variations between 20\%, when \( \gamma \) changes from 0.7 to 1, and almost 50\% when the stiffness parameter changes from 0.3 to 0.7.

A first experiment was performed to test the MUSMAR response to plant dynamic changes. In this experiment the freezing and change detection mechanisms are not active since the aim is to use it as a standard for comparisons with other situation. Figures 5 up to 7 show the results. As can be seen, Fig. 6 and Table 1, there are strong transients at the instants in which the value of the stiffness parameter is changed.

In a second experiment, the freezing gains and change detection mechanisms were used and the control signal was bounded, i.e., \(|u(t)| \leq 0.5\). The results are shown in Figures 8 up to 10. All the simulations results were obtained setting \( g^*(t) = 0 \) and \( \{w(t)\} \) a uniformly distributed random sequence over \([-10^{-3}, 10^{-3}]\).
The following set of MUSMAR parameters were chosen for both experiments: $T = 11$, $n = 4$, $m = 3$, $\lambda = 0.99$, $\rho = 1$ and $\sigma_n^2 = 0.00001$. In what concerns the parameters of the controller gains freezer and change detection mechanism these are: $\epsilon = 0.001$, $k = 3.5$ and $T_u = 2000s$. For the computation of $\hat{y}(t)$, $\sigma(t)$ and $\hat{F}(t)$ two moving windows were used, one with the last $120$ output values, and the other one with the last $10$ vectors of gains.

The results obtained in the experiments show an improvement in the performance of MUSMAR-CGDF due to the inclusion of the change detection and gains freezing mechanisms. This improvement is quite notorious in regions where the system dynamics changes occurred, i.e. at $4000s$ and $8000s$ where for less control effort better transients are obtained, compare Figs. 6 and 9 and Figs. 5 and 8, see also Table 1. An important aspect is the fast detection, see Table 5, which together with the covariance matrix resetting enhances the re-tuning of the gains and the convergence to optimal gains, allowing this way a faster readaptation to the new situation. In steady-state, to the same control effort, MUSMAR-CGDF shows a better dynamical behavior, i.e., after the gains freezing the plant output amplitude is smaller, nearly $10^{-3}$ against $10^{-2}$ obtained in the first experiment. To this fact it is not strange that the dither noise is removed when the gains are freezed.

### Table 1. Maximum values for $|y(t)|$ and $|u(t)|$ at both experiments

<table>
<thead>
<tr>
<th>Time Interval</th>
<th>$y(t)$</th>
<th>$u(t)$</th>
<th>$y(t)$</th>
<th>$u(t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$[0, 4000]$</td>
<td>1.3</td>
<td>1.71</td>
<td>0.3</td>
<td>0.6</td>
</tr>
<tr>
<td>$[4000, 8000]$</td>
<td>43</td>
<td>62</td>
<td>0.3</td>
<td>0.6</td>
</tr>
<tr>
<td>$[8000, 12000]$</td>
<td>2.1</td>
<td>3</td>
<td>0.15</td>
<td>0.15</td>
</tr>
<tr>
<td>$[12000, 16000]$</td>
<td>0.05</td>
<td>0.045</td>
<td>0.04</td>
<td>0.06</td>
</tr>
</tbody>
</table>

### Table 2. $\gamma$’s change, Gains Freezing and Change detection

<table>
<thead>
<tr>
<th>$\gamma$’s change(sec)</th>
<th>$\gamma = 0.3$</th>
<th>$\gamma = 0.7$</th>
<th>$\gamma = 1$</th>
<th>$\gamma = 0.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Detection(sec)</td>
<td>4000</td>
<td>8000</td>
<td>8018.5</td>
<td>12000</td>
</tr>
<tr>
<td>Freezing(sec)</td>
<td>2023</td>
<td>6075</td>
<td>10123</td>
<td>14031</td>
</tr>
</tbody>
</table>

### 6. CONCLUSIONS.

The results presented in this paper suggest that MUSMAR might be used with advantage as a controller gain tuner. Whenever a change in dynamics is detected according to some criterion, the adaptation mechanism embodied in MUSMAR becomes active to re-tune the gains, being stopped otherwise. This idea have a practical value because a current objection placed to adaptive control by industrial practitioners is the possibility of gains drifting continuously. MUSMAR provides thus a powerful tuning tool without this drawback.
Appendix - Proof of Proposition 1

According to the control strategy used

\[ g(t + i) \approx \theta_i(F_0) \eta(t) + H_i(F_0, F)s(t) \quad (17) \]

\[ u(t + i - 1) \approx \mu_{i-1}(F_0) \eta(t) + G_{i-1}(F_0, F)s(t) \quad (18) \]
in which

\[ H_i(F_0, F) = \psi_i(F_0) + \theta_i(F_0)F \quad (19) \]

\[ G_{i-1}(F_0, F) = \phi_{i-1}(F_0) + \mu_{i-1}(F_0)F \quad (20) \]

Let \( F \) be computed according to (8) and (9). By adding and subtracting \( \theta_i^2(F_0)F_0 \) and \( \mu_i^2(F_0)F_0 \), this becomes

\[
F = -\frac{1}{\alpha(F_0)} \left( \sum_{i=1}^{T} \theta_i(F_0)H_i(F_0, F_0) + \right. \\
\left. + \rho \sum_{i=1}^{T-1} \mu_i(F_0)G_i(F_0, F_0) + \rho F_0 \right) + \\
\left. + \frac{F_0}{\alpha(F_0)} \left( \sum_{i=1}^{T} \theta_i^2(F_0) + \rho \left( \sum_{i=1}^{T-1} \mu_i^2(F_0) \right) \right) \right)
\]

(21)

If \( \rho F_0 / \alpha(F_0) \) is added and subtracted then (21) becomes

\[
F = -\frac{1}{\alpha(F_0)} \left( \sum_{i=1}^{T} \theta_i(F_0)H_i(F_0, F_0) + \right. \\
\left. + \rho \sum_{i=1}^{T-1} \mu_i(F_0)G_i(F_0, F_0) + \rho F_0 \right) + \\
\left. + \frac{F_0}{\alpha(F_0)} \left( \sum_{i=1}^{T} \theta_i^2(F_0) + \rho (1 + \sum_{i=1}^{T-1} \mu_i^2(F_0)) \right) \right)
\]

(22)

Now we are going to turn our attention to the gradient of the cost function \( J_T(t) \), defined in (2). This is given by

\[
\frac{1}{2} \frac{\partial J}{\partial F} = E \left[ \sum_{i=1}^{T} g(t + i) \frac{\partial g(t + i)}{\partial F} \right] + \\
\rho u(t + j - 1) \frac{\partial u(t + i - 1)}{\partial F}
\]

(23)

or by

\[
\frac{1}{2} R_{ys}(i) \triangleq E[g(t + i)s(t)] = R_s H_i(F_0, F)
\]

(25)

R_{ys}(i) \triangleq E[g(t + i)s(t)] = R_s H_i(F_0, F)

(26)

Since \( \mu_0 = 1, \phi_0 = 0 \) and \( G_0(F_0, F_0) = F_0 \) it follows from (24) and (22) that

\[
F = F_0 - \frac{1}{2\alpha(F_0)} R_s^{-1} \frac{\partial J(F_0)}{\partial F}
\]

(27)

7. REFERENCES


