Quadrature Van der Pol Oscillators Using Second Harmonic Coupling

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Abstract — In this paper, a new coupling mechanism is used to synchronize two Van der Pol oscillators. This coupling is established using the second harmonic appearing in the common mode current of each oscillator. The common mode current is measured by a current mirror, and is amplified by a current amplifier. The amplifier introduces negative feedback, so that the current in the current mirror measuring diode of the first oscillator is nearly equal to the common mode current of the second oscillator; hence the coupling is established. It is shown that the system of two oscillators is described by two differential equations where the coefficients in one equation have the perturbations defined by the second oscillators and vice versa. The coupling amplifier gain is defined. The developed concepts are demonstrated on a 5 GHz CMOS LC oscillator with quadrature outputs. The oscillator phase noise is lower than -116 dBc/Hz at 1-MHz offset.

I. INTRODUCTION

The problem of generating RF sinusoidal signals in quadrature can be formulated as follows [1]: There are two nonlinear systems each exhibiting self sustained periodic oscillations, these two systems do interact through coupling. An emphasis is placed on selecting the technique, type and strength of coupling in order to generate accurate quadrature signals.

Several techniques to generate quadrature signals have been presented in literature [2,3,4]. These coupling techniques can be implemented using either passive [2] or active [3] networks. A VCO followed by RC networks was used in [2] for quadrature generation. Transistors were used as coupling circuits in [3] to force two oscillators to operate in quadrature. The former technique suffers from the need to include buffers between the VCO and the RC network [4]. While the latter technique have the drawback of increased power consumption and the existence of a trade off between phase noise and quadrature generation. The different coupling techniques were compared in [4].

Coupling modes can be established using odd or even harmonics. The amplitude stabilization results in currents with rich harmonic content. The differential current component is close to sinusoidal, with the frequency equal to the oscillator oscillation frequency, the common mode current component is also close to sinusoidal, but its frequency is equal to double the oscillation frequency of oscillator. For synchronization, the differential current component provided by the synchronization circuit should coincide in phase with the current supplied to the parallel LC-circuit by the amplitude stabilization circuit. Then the synchronizing circuit will partially substitute the amplitude stabilization circuit, and this mechanism is needed for synchronization to occur. The common mode component may also be used for synchronization, but in this case one have to use the equivalent second harmonic in the common mode output. Most of the coupling types presented in literature were realized using the first or odd harmonics. The second harmonic common mode coupling type was presented in [4]. The coupling was realized using passive narrow band networks. One drawback of such realization is the extra die area required to implement the passive devices. The approach of coupling using the first and second harmonics simultaneously was verified designing a low power, low phase noise 5 GHz LC oscillator with quadrature outputs in [5]. The benefits of using simultaneous odd and even harmonic coupling is discussed in [5]. For quadrature generation, the self-oscillating nonlinear systems presented in literature included either RC oscillators [6] or Robinson type oscillators. Both types of oscillators were compared in [6].

This paper presents a new coupling technique realized using the second harmonic coupling type to enforce quadrature relation between two Van der Pol (VDP) oscillators. In Part II, the circuit implementation and the conditions of establishing a strong second harmonic component in the common mode current are derived. Part III presents the simulation results and highlights the benefits and drawbacks of the new coupling technique. Finally in Part IV we draw some conclusions.

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II. SYNCHRONIZATION THEORY

We consider that each individual oscillator (Fig. 1) has a nonlinear element realized by transistors \( M_1 \) and \( M_2 \). This element is described by the input-output characteristic

\[
    i_d = -2B_N(V_{DD} - V_{TN})v_d (1 - \frac{v_d^2}{4V_{DD}^2}) \tag{1}
\]

where \( v_d = v_1 - v_2 \) and \( i_d = i_1 - i_2 \). Here \( B_N = (\mu_n C_{ox} / 2)(W / L) \), \( V_{DD} \) is the power supply voltage, and \( V_{TN} \) is the threshold voltage of n-channel transistors. It is possible to show that with this type of the characteristic the nonlinear element provides a common mode current

\[
    i_c = (B_N / 2)v_d^2 \tag{2}
\]

where \( i_c = i_1 + i_2 \). The result (2) does not include the dc component \( 2B_N(V_{DD} - V_{TN})^2 \) and does not describe the falling part of the dependence \( i_c = f_c(v_d) \) (this falling part is important for evaluating the stability of coupling using second harmonic, and this problem is not considered in this paper).

![Figure 2. VDP oscillator equivalent circuit](image)

For the standard equivalent circuit of VDP oscillator (Fig. 2) one can write the equation

\[
    \frac{1}{C} \int i_{Cd} dt = L \frac{di_{Ld}}{dt} + R_s i_{Ld} \tag{3}
\]

Using approximation \( L \frac{di_{Ld}}{dt} = v_d \) and introducing the new variable \( \ddot{v} = \omega_0 v_d \) one can transform (3) into equation

\[
    \ddot{v} + \omega_0^2 v = -\omega_0^2 R_s C \ddot{v} - \omega_0^3 L i_d \tag{4}
\]

Substituting (1) in (4) one obtains that

\[
    \ddot{v} + \omega_0^2 v = -\omega_0^2 R_s C \ddot{v} + \omega_0^3 \left[ 2B_N(V_{DD} - V_{TN}) \ddot{v} \left( 1 - \frac{v_d^2}{4\omega_0^2 V_{DD}^2} \right) \right] \tag{5}
\]

Two VDP oscillators synchronized by the second harmonic are shown in Fig. 3. The synchronization circuit consists of the measuring current mirrors introduced between power supply and the coil tap in each oscillator and the coupling differential current amplifier with the gain of \( A_i \). The synchronization circuit introduces small variation of the power supply voltage in each oscillator by the coupling current \( i_{cpl} \) (or, correspondingly, by \( i_{cplR} \) ) provided from the second oscillator. This small power supply voltage variation modifies the coefficients of the differential equation (5) and this mechanism results in the synchronization of the oscillators. In this paper we restrict ourselves to the demonstration that this coupling exists, i.e. that one can find the relationships between the oscillator parameters (basically we show that one can find suitable amplifier current gain) that two differential equation with variable parameters are satisfied in steady-state operation. The formal derivation of the solution, and verification of its stability will be demonstrated during conference presentation.

![Figure 3. Synchronization of two VDP oscillator](image)

We will describe the left oscillator by the variable \( v_d = v_1 - v_2 \) and the right oscillator by \( u_d = u_1 - u_2 \). First, we notice that the coupling currents and the common mode currents of oscillators are described by the equations

\[
    \begin{align*}
    (i_{cl} &+ i_{cpl}) A_i = i_{cplR} \\
    (i_{cr} &+ i_{cplR}) A_i = i_{cplL} \tag{6}
    \end{align*}
\]

From (6) one can find, for example, that with reasonably high \( A_i \) the coupling current \( i_{cplL} = -i_{cl} - (i_{cr} / A_i) \) and the current of the measuring diode is \( i_L = -i_{cr} / A_i \). Thus, the amplifier introduces the feedback that cancels the common mode current from the oscillator where this measuring diode is connected, yet allows flowing in this diode the current of another oscillator. This is exactly what is required for coupling.

To obtain the differential equation of the left oscillator we have to substitute in (5), instead of \( V_{DD} \), the expression

\[
    \frac{V_{DD} - |V_{TP}|}{g_d A_i} = \frac{V_{DD} - |V_{TP}| - B_N u_d^2}{2 g_d A_i} \tag{7}
\]
where $1/g_d$ is the resistance of the measuring diode connected transistor (expression (7) has a small error, it is better to use $V_{SG}$ voltage and not $|V_{TP}|$ but this will only complicate the derivation). In the steady-state oscillation when $u_d = U_d \omega_0 \cos \omega_0 t$ this gives us the differential equation

$$\ddot{v} + \omega_0^2 v = -\omega_0^2 R_S C \dot{v}$$

$$+ \omega_0^2 L \left[ 1 - \frac{\dot{v}^2}{4 \omega_0^2 (V_{DD} - |V_{TP}|)^2 (1 \pm b U_d^2 \cos 2\omega_0 t)} \right]$$

Introducing the notations $G = 2B_N (V_{DD} - V_{TN} - |V_{TP}|)$, $a = B_N / (4g_d A_i (V_{DD} - V_{TN} - |V_{TP}|))$, and $b = B_N / (4g_d A_i (V_{DD} - |V_{TP}|))$, one can rewrite (8) in a more convenient form

$$\ddot{v} + \omega_0^2 v = -\omega_0^2 R_S C \dot{v} + \omega_0^2 L G \dot{v}(1 \pm a U_d^2 \cos 2\omega_0 t)$$

$$\left[ 1 - \frac{\dot{v}^2}{4 \omega_0^2 (V_{DD} - |V_{TP}|)^2 (1 \pm b U_d^2 \cos 2\omega_0 t)^2} \right]$$

Considering that the terms $a U_d^2 \cos 2\omega_0 t$ and $b U_d^2 \cos 2\omega_0 t$ are small, one rewrites (9) as

$$\ddot{v} + \omega_0^2 v = -\omega_0^2 R_S C \dot{v} + \omega_0^2 L G \dot{v}(1 \pm a U_d^2 \cos 2\omega_0 t)$$

$$\left[ 1 - \frac{\dot{v}^2}{4 \omega_0^2 (V_{DD} - |V_{TP}|)^2} \right]$$

or, finally as

$$\ddot{v} + \omega_0^2 v = -\omega_0^2 R_S C \dot{v}$$

$$+ \omega_0^2 L G \dot{v}(\dot{v}^2 [1 \pm (\pm b \mp a) U_d^2 \cos 2\omega_0 t]$$

$$\left[ 1 - \frac{\dot{v}^2}{4 \omega_0^2 (V_{DD} - |V_{TP}|)^2} \right]$$

Similar transformations and approximations can be done for the second oscillator. With coupling, the second oscillator will be described by the equation

$$\ddot{u} + \omega_0^2 u = -\omega_0^2 R_S C \dot{u}$$

$$+ \omega_0^2 L G \dot{u}\left[ 1 \pm a U_d^2 \cos 2\omega_0 t \right.$$
The calculation shows that (18) results in very moderate values of the gain.

III. CIRCUIT IMPLEMENTATION

The previous theoretical analysis indicates that the second harmonic can be used for synchronization of two LC oscillators. In order to validate the theory we simulated the oscillator shown in Fig. 3. The conventional first harmonic coupling circuit (not shown in this figure) was first used to establish quadrature. Then this coupling circuit was disconnected, leaving the full synchronization load carried by the second harmonic coupling circuit.

The circuit was designed for the 0.18 \( \mu \text{m} \) CMOS technology, for an oscillation frequency of 5 GHz. The circuit parameters are the following. The second harmonic synchronization transistors have (W/L) = 10 \( \mu \)m / 0.18 \( \mu \)m. The current sensing transistors have the dimensions (W/L) = 4 \( \mu \)m / 0.18 \( \mu \)m. The core transistors forming active nonlinear circuit have the dimensions of (W/L) = 100 \( \mu \)m / 0.18 \( \mu \)m. The second harmonic coupling transistors are biased by a tail current equal to 100 \( \mu \)A. The circuit supply voltage is 1.8V and draws a current of 1.2 mA.

IV. SIMULATION RESULTS

The oscillator phase noise at 1 MHz offset is lower than -116 dBc/Hz as shown in Fig. 4. This phase noise is 5 dB higher than the phase noise obtained in [5]. The degradation in phase noise might be offset by the benefit that the die area is spared by not using passive elements in the coupling circuit. A plot of the quadrature outputs at 5 GHz is shown in Fig. 5.

V. CONCLUSION

Synchronization of LC oscillators usually requires strong coupling, entailing the use of high bias currents in the coupling circuits. Excluding the synchronizing circuit with high bias current results in the minimal shift in the oscillation frequency compared to the oscillation frequency of the standalone oscillator. Hence the degradation in oscillator phase noise can be minimized. Coupling using the second harmonic does not create any additional load on each oscillator. An advantage of the proposed coupling mechanism is that the area consuming passive coupling circuits are not used as well. The conventional coupling circuit of any type may be disconnected once the second harmonic coupling is realized.

A new type of coupling circuits using second harmonic is presented. The coupling circuit is realized using active elements. A theory was developed describing the necessary condition in order to obtain stable common mode second harmonic coupling. These conclusions are confirmed by the design of a 5 GHz low power quadrature oscillator using second harmonic common mode coupling. The circuit has a simulated phase noise of –116 dBc/Hz at 1 MHz offset.

REFERENCES