OFDM Communication System for PLC: Equalization Approaches

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Abstract — An OFDM communication system suitable for power line communication (PLC) is described. It uses an interleaver/deinterleaver, a LDPC (Low Density Parity Check code) encoder/decoder and an equalizer. Two different algorithms were used and compared in the equalizer: the Fast Kalman and the NLMS (Normalized Least Mean Squares) algorithm. Simulation results regarding several different transmission conditions show that the Fast Kalman receiver is preferable over the NLMS one in most of the cases. The performance of the LDPC encoder/decoder is also analysed.

Keywords – OFDM, Equalizers, Fast Kalman, NLMS, PLC, LDPC

I. INTRODUCTION

Nowadays, one of the most important challenge is to design adaptive communication systems, which deal with errors very efficiently, namely through the use of powerful equalizers and error correcting codes. Also, they should have a high adaptability to varying channels.

The Orthogonal Frequency Division Multiplexing (OFDM) is a multicarrier modulation scheme, which suits very well the characteristics stated above and is used in most of the present communication standards, either for cable or wireless communication. OFDM has several advantages such as low complexity equalization in dispersive channels, robustness to interferences, mainly narrowband interference avoidance (NBIA) [1] and spectral density scalability/adaptability (e.g., adaptive bit loading [2]). For instance, a recent standard employing MB-OFDM UWB for very high data rates (53.3, 80, 106.7, 160, 200, 320, 400, and 480 Mb/s) is WiMedia, a radio platform standard for short-range, high-speed wireless connectivity. Also, a multicarrier approach, like OFDM, has proved to be suitable for high-speed wireless communications: Combining OFDM with two-dimensional spreading (time and frequency domain spreading), an orthogonal frequency- and code-division multiplexing (OFCDM) system was considered for the downlink transmission in future 4G networks [3].

Due to the multipath and impulsive noise effects of the power line channel, OFDM is widely used in broadband power line communication (PLC). It is very robust against narrow band interferences and frequency selective fading. The MB-OFDM proposal once submitted to the IEEE 802.15.3a standards group can also be applied to power line channel, to achieve broadband high data rate transmission. In [4] a very interesting study of high data rate PLC above 30 MHz bandwidth is presented. It has been shown that the power line channel is suitable for broadband UWB over 50 - 550 MHz frequency band and that higher transmission rate can be achieved by exploiting the frequency range above 30 MHz.

In this paper we present a global OFDM communication system suitable for using in PLC. The goal is to evaluate the performance of the system when using two different equalizers in the receiver: a Fast Kalman equalizer or a NLMS one [5]. Also, the influence of a error correcting code, namely a low density parity check code (LDPC) is evaluated. Based on the obtained results, one can proceed to the implementation process, either using FPGA or hybrid technology.

In Sec. II the global OFDM communication system is described and its functioning is explained. In Sec. III simulation results regarding the communication system under different transmission conditions are presented, with the main goal of characterizing the system’s performance with each equalizer, in order to compare both solutions. Sec. IV is dedicated to evaluate the encoder/decoder performance and its influence in the communication system. Finally, in Sec. V conclusions are addressed.

II. SYSTEM ARCHITECTURE

The developed OFDM Communication System is composed by the Emitter, Channel and Receiver blocks presented next.

A. Emitter

Bit Source - It creates a sequence of random bits, uniformly distributed. The implemented bit source creates a frame with 320 message bits (32 sub-frames with 10 message bits each), with period \( T_s \). This period is described by the following expression:

\[
T_s = \left(\frac{1}{F_s}\right) \times (N + N_c).
\]

Where \( F_s \) (12.5 MHz) is the system sampling frequency, \( N \) (256) represents the dimension of the IFFT and \( N_c \) is the dimension of the cyclic prefix.

Encoder - After the creation of the information frame (with 320 bits), each one of the 32 words will have 10 bits. This frame is sent to the Low Density Parity Check (LDPC) encoder, which generates a 512
bits frame. So, before the sub-frame processing, it is created a regular sparse parity check matrix, $H$ (for use in LDPC codes), with 6 rows by 16 columns. $H$ is a regular matrix, which means that it has a constant number of ‘ones’ per column and per rows. The $H$ matrix implemented has a row weight of 8, and a column weight of 3. After the creation of the $H$ matrix, it is determined the generator matrix $G$, for the creation of the code word. It has the dimensions of 10 rows by 16 columns. The codewords, with a 16 bit length, are created according to the following expression: $C = mG$. As there is 32 words, with 10 message bits each, the encoder will create 32 words of 16 bits each (10 message bits plus 6 parity bits). So, the frame has now a length $32 \times 16 = 512$ bits.

**Interleaver/Deinterleaver** - The interleaver has 32 rows by 16 columns and is responsible for the frame’s bit interleaving. Information is written in row wise, and read out column wise [6] [7].

**QPSK Modulation** - The previous frame (with 512 bits) is sent to the QPSK modulator, which creates a 256 symbols frame. This modulator uses the Gray coding. It has incorporated a gain with the value $\sqrt{2}$, to produce unitary output signal power.

**IFFT Signal Multiplexing** - The QPSK frame is sent to the IFFT block. The IFFT block has 256 coefficients for input and permits 256 samples in its output. Consequently, the OFDM signal will have 256 sub-carriers. Similarly to the QPSK modulator, the IFFT also has a gain, with the value $\sqrt{N}$ (where $N$ is the IFFT’s dimension, and has the value 256), to maintain unitary signal output power.

**Cyclic Prefix Insertion** - The block cyclic prefix receives the multiplexed signal from the IFFT. It copies the last $N_g$ samples of the frame at the IFFT output, which forms the OFDM symbol, and concatenate with the $N$ samples that form the output frame of the IFFT. The purpose of the cyclic prefix is to reduce the ISI [7] [8]. Its dimension should be as small as possible, due to bandwidth efficiency. To compensate the signal delays imposed by the channel, the cyclic prefix should have a time length greater than the channel impulse response. The used power line channel [9], has an amplitude and impulse response (with a length of 1.5 x 10^{-6} s) depicted in Figures 2 and 3, respectively. Consequently, the number of samples of the cyclic prefix should satisfy the following condition: $\left(\frac{N_g}{F_s}\right) \geq 1.5\mu s$. As $F_s$ has the value 12.5MHz, the value chosen to $N$ was 32, $1/8$ of the frame length. $N_g$ is the minimum value that compensates the signal delays due to the impulse channel response. The frame at the output of the cyclic prefix will have, a length $N + N_g = 256 + 32 = 288$ samples.

**Interpolation** - Before the signal’s modulation and transmission, it must be interpolated. It was used an interpolating factor $L = 8$ to perform the pass-band shift of the OFDM signal, before it’s transmission to the channel. The interpolation process has two steps. First, the input is interpolated to a higher frequency, through the insertion of $L - 1$ zeros between samples [8]. Afterwards, the output signal is filtered. The frame at the interpolation block’s output has a length of 2304 samples.

**Modulator** - Before the OFDM signal is transmitted to the channel, it should be shifted to pass-band. This block multiplies the real part of the input sample by the factor $\cos(2\pi f_c t)$, and the imaginary part by $-\sin(2\pi f_c t)$, where $f_c = 12.5$ MHz. After that operation, both parts are added. The frequency $f_c$ should be less or equal than the cutoff frequency of the filter at the receiver’s input, to maintain the information.

**B. Power Line Channel**

In Figures 1 and 2, is depicted the channel’s amplitude characteristic, and the impulse response, respectively.

![Channel’s frequency response: amplitude characteristic.](image1)

![Channel’s impulse response.](image2)

It was created a power line channel, defined by Gotz, Rapp and Dostert [9], of type II. That type of power line channels are considered good channels, has 6 branches, and links of approximately 110 meters.
C. Receiver

Demodulator - The signal from the channel is shifted from pass-band to base-band. To perform that operation, the received signal is multiplied by \(\exp(-j2\pi f_t t)\) and so, shifted to base-band.

Decimation - After being demodulated, the signal should be decimated to achieve its original sampling frequency. The block decimation downsamples the input signal in discrete time domain, with a frequency \(K\) times lower than its frequency. The constant \(K\) is denominated the decimating factor, and it has the same value than the interpolating factor \(L\), namely 8 [8]. Similarly to the interpolating process, the decimation process is made in two steps. First, the signal is filtered by a FIR filter in the direct form. The second step consists of decimating the filtered signal to a lower frequency, discarding \(K-1\) consecutive samples and keeping the next. The frame at the decimation block’s output will have 288 samples.

Cyclic Prefix Removal - After the decimation process, the signal’s cyclic prefix is removed. From the set of \(N+N_\pi\) received samples (288), the first \(N_\pi\) samples are discarded, thus a 256 symbol frame is recovered.

FFT Signal Demultiplexing - After the signal’s cyclic prefix removal, the OFDM symbol should be demultiplexed and consequently transformed in a 256 QPSK symbol frame. To perform that operation, the FFT block is used. It also has a gain, with value \(1/\sqrt{N}\) (where \(N = 256\)), whose objective is to permit an unitary signal power at the FFT’s output.

Fast Kalman Equalizer - In 1978, Ljung, Morf and Falconer developed a fast RLS (Recursive Least Square) algorithm, which reduce substantially the performed calculus. The resulting algorithm’s (called the Fast Kalman Algorithm) operations is proportional to \(N\), instead of \(N^2\). It also maintains the fast convergence speed. The goal of the Fast Kalman algorithm is to reduce the number of multiplications and additions, thus increasing its convergence speed. To achieve that purpose the Kalman gain, \(k[n]\) is calculated in a different way when compared to the Kalman Filter algorithm [5].

NLMS Equalizer - LMS coefficients updating is given by [5]:

\[
c(n+1) = c(n) + 2\mu e(n)a(n)
\]

The adjustment term \(2\mu e(n)a(n)\), contains the input vector \(a(n)\), which implies that the convergence speed of the algorithm depends on the signal input power. One way of the algorithm convergence become more independent from the input signal is to change the adjustment term through a term inversely proportional to an important characteristic of the input signal (absolute value, power). In the normalized LMS algorithm, the adaptation step is normalized relatively the square of the Euclidean norm of \(a(n)\). The consequent update equation is:

\[
c(n+1) = c(n) + \frac{2\mu}{b + \|e(n)\|^2} e(n)a(n)
\]

Where the adaptation step \(\mu\), is a constant which can take any value between 0 and 1, to assure convergence of the algorithm. In the NLMS, the choice of the \(\mu\) parameter is less relevant than in the LMS. In the LMS algorithm, the possible set of values for \(\mu\) depends from the power of the input signal, which can not be known exactly. The normalization has many advantages, because it can deal with a wide set of signal powers, in a balanced way. The number of coefficients used in the NLMS algorithm is 256, as the Fast Kalman’s algorithm.

QPSK Demodulator - This block incorporates a “symbol decisor”, which decides the “correct” symbol from the equalizer’s received symbol. It is also responsible for performing the QPSK demapping of the signal from the equalizer, transforming the 256 symbol input frame into a 512 bits output one. So, this block performs the “QPSK Modulator” inverse processing.

Deinterleaver - The 512 bits frame from the QPSK demapping is sent to the deinterleaver’s matrix (32 rows by 16 columns), to deinterleave the bits. Afterwards, the frame is sent to the decoder.

Decoder - LDPC Codes are considered one of the best error correcting codes nowadays. A LDPC code is a binary block code characterized by a sparse parity check matrix, \(H\). The \(H\)’s rows are the representation of the parity equations, and the columns represent the bits of the code word. The element of row \(j\) and column \(i\) is ‘1’, if the bit \(x_i\) belongs to the \(j^{th}\) parity check equation. If the matrix \(H\) has a constant number of ‘1’s per row an per column, it is called a regular parity check matrix \(H\). In this work it is used a regular 6x16 matrix \(H\), with a row weight of 8, and a column weight of 3. Each 16 bit’s received code word is sent to the LDPC decoder, which with knowledge of the parity check sparse matrix \(H\), verifies if the code word belongs to the codeword set and correct it, if necessary. Afterwards, the message bits are extracted (in the data receiver) from the corrected codeword and are sent to the receiver.

Data Receiver – This last block removes the 32 words of 10 message bits each, from the 512 bits frame, recovering the data.

III. SIMULATION RESULTS

The simulation’s focus is mainly on the equalizer’s efficiency, and on the equalizer’s algorithm comparison (Fast Kalman and NLMS), because, as stated before, the equalizer is an essential block to contribute to the
OFDM system’s efficiency. The behaviour of the encoder/decoder is also evaluated.

To compare the equalizer’s (Fast Kalman and NLMS) behaviour, 4 sets of simulations are made: A) System without power line channel (Ideal Channel); B) System with power line channel and no noise; C) System with noisy ideal channel; D) System with power line channel and noise. All the presented results respect to the faster convergence behaviour still maintaining the equalizers’ stability.

A. System without Power Line Channel (Ideal Channel)

Here, the goal is to characterize the equalizer’s behaviour in the presence of no channel at all, which is the ideal channel condition. The sequence of the pilot bits used is a frame of 512 bits with the value ‘1’.

In the Fast Kalman algorithm, the expected result is the first coefficient with the value ‘1’, and the other coefficients with the value ‘0’. As depicted in Figure 3, the coefficient’s value is attained extremely quickly. And, as expected, the first coefficient has the value ‘1’, and all the others had attained the value ‘0’ (approximately).

In opposite to the Fast Kalman result, some coefficients of the NLMS will attain values different from ‘1’ and ‘0’, as depicted in Figure 4.

The Fast Kalman error signal goes to ‘0’ very abruptly, which indicates a high convergence speed, as depicted in Figure 5. For the NLMS, the evolution to ‘0’ of the error signal is not so fast when compared to the Fast Kalman case, as depicted in Figure 5.

B. System with Power Line Channel and no Noise

Here the goal is to characterize the equalizer’s behaviour in the presence of a system already with a power line channel, but a good one – with no noise. The results are presented in Figures 6 to 8. Comparing the results of Figures 6 to 8 with those of Figures 4 and 5, respectively, the following conclusions can be taken:

- The evolution of the coefficients with power line is not smooth as the result obtained without it.
- Almost all the Fast Kalman’s coefficients (256) are now used in Figure 6, in opposite to what happened in Figure 3, where practically only the first coefficient is needed. In the NLMS case, also all the coefficients were used again.
- Both the error signal evolutions are less monotonic when compared to those without power line, as can be seen from Figures 5 and 8. As depicted in Figure 8, the error signal is approximately ‘0’ after the 100th sample for the Fast Kalman case and after the 175th sample for the NLMS case.
C. System with Noisy Ideal Channel

This case characterizes the noise reduction of a system without the power line channel, but with noise, namely SNR = 6 dB. Once the most recent OFDM systems are adaptive, one may avoid to communicate in very noisy sub-channels, so, we discarded the most severe (and typical) impulsive noise, and used a most generic Gaussian noise, common to a large number of channels. The results are presented in Figures 9 to 11.

When comparing these results with case A. (ideal channel and no noise), we can conclude that both equalizers have identical but small deterioration with the introduction of noise. Also, the Fast Kalman’s error signal evolution has a high convergence speed, when compared with the NLMS’s error signal evolution.

D. System with Power Line Channel and Noise

This case characterizes the noise reduction of a system with a noisy power line channel, with SNR of 6 and 10 dB. The results are presented in Figures 12 to 14.
Comparing these results with all those presented before we can conclude the following:

- The Fast Kalman receiver converges always (much) faster than the NLMS one.
- Although both systems deteriorate their performance as the SNR decrease (as expected), the Fast Kalman still presents the better results.
- As a RLS algorithm, the Fast Kalman beneficiates of a more reduced final misadjustment than the NLMS, which, as well known, presents a final misadjustment dependent on the autocorrelation matrix eigenvalues.

IV. ENCODER/DECODER PERFORMANCE

Here, we analyse the encoder/decoder blocks influence. The result is depicted in Figure 15.

In this test a 500 bits frame was transmitted varying the SNR. In each iteration, the input frame was compared with the output frame, noticing the bits with error and performing the error percentage. As depicted in Figure 15, with at least a SNR of 1dB, it is possible to attain a negligible error bit.

V. CONCLUSIONS

In this paper a complete OFDM communication system suitable for PLC is described. Two equalizer types were considered in the receiver: a Fast Kalman and a NLMS one. From the simulation results considering several conditions in the transmission environment we can conclude that the Fast Kalman solution has a (much) faster convergence than the NLMS one, in all the tested conditions. It is important to refer that although it presents a higher computational effort regarding the NLMS one, the total increase in the receiver’s computational effort is only 0.25% more than the NLMS case. So, the Fast Kalman solution is preferable, especially in high rate varying channels, as is the case with power lines. Also, the behaviour of an LDPC error correcting code is determinant in the system performance. It was verified that errors can be drastically reduced by using suitable SNRs for the chosen code. For the used LDPC code (16 bit regular code) the edge was a SNR ≈ 1 dB.

The presented results were obtained using a PLC environment, but they can also be extended to other OFDM applications.

REFERENCES