On-line model bank enlargement for multiple model adaptive control

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Abstract—The switched multiple model adaptive control is a strategy that allows to tackle the control of plants whose parameters present sudden variations of possibly wide amplitude. It consists of a bank of local controllers that are applied to the plant according to the decision made by a supervisor. In turn, the supervisor relies on a bank of local plant models that cover the possible outcomes of plant dynamics. The problem of adding new pairs of models/controllers to the existing bank, based on on-line data, is considered. A decision mechanism for achieving this is explained and the attainable increase in performance is illustrated with a case study on the control of a flexible transmission with abrupt load changes.

I. INTRODUCTION

The switched multiple model adaptive control (SMMAC) is a strategy designed to tackle the problem of controlling plants with sudden changes in plant dynamics [1], [2]. These ruptures in dynamics may be yielded either by abrupt variations of plant model parameters (induced, e.g., by load or structural changes) or by large amplitude set-point changes in nonlinear plants.

In the most basic version, SMMAC consists of a bank of (usually linear) controllers that covers the possible plant dynamics outcomes. During each sampling interval, the controller whose output is actually applied to the plant is selected by a supervisor made up by a bank of models, each one matching a controller in the controller’s bank (hereafter referred as the local models/controllers). This selection is done by evaluating the model whose output best fits the actual plant output and choosing the corresponding controller. A dwell time strategy is followed to ensure stability: Accordingly, once a controller is applied to the plant, it remains so for a minimum interval of time (the dwell time).

This basic structure has been the subject of several extensions and studies [3], [4], [5], [6], [7], [8]. In particular, it has been recognized that the inclusion of an adaptive self-tuning controller in parallel with the bank of fixed controllers has a significant advantage [8]. Indeed, while the fixed controllers tackle the transients due to abrupt changes of dynamics, the self-tuning channel increases the performance in the long run, by fine tuning on the exact value of the plant parameters.

An open issue is the management of the model/controller using on-line data. In particular, starting with a reduced number of models/controllers, how is it possible to add new ones such as to increase the performance when these dynamics (a priori unexisting in the banks) are revisited?

The contribution of the present paper consists in a learning algorithm to solve this problem. Departing from a limited set, the bank of local controllers is successively enlarged on-line when new dynamics are observed, by using controllers to which the self-tuner has converged. For this scheme to work properly, new controllers are only added to the bank if some criteria related to the difference with respect to existing controllers are fulfilled. The proposed structure has the advantage of avoiding repeated adaptation transients when the plant parameters reach values not considered a priori.

The predictive adaptive MUSMAR algorithm is used [9] in the self-tuner channel. This controller, for which a number of successful industrial applications are reported [10], [11], [12] was found suitable for the current problem, although by no means unique. The MUSMAR algorithm provides an approximation to the optimal LQ controller constrained to the imposed controller complexity.

The proposed SMMAC with learning is illustrated in the flexible transmission system introduced in [13] as a benchmark problem. The control of this plant with SMMAC including a self-tuning channel was thoroughly studied in [14], [15]. As explained above, the novelty of the present work is to exploit the adaptive channel, together with convenient supervisor rules, to build on-line the local models/controllers bank in order to increase the controlled system’s performance.

The paper is organized as follows: after the introduction (this section) the methodology of control using multiple models is discussed. In section 3 more details about the learning algorithm coupled with Multiple Model Switched Adaptive Control structure are presented. The local controllers design is presented in section 4. In section 5, simulation examples are given to illustrate the advantages of using learning with multiple models for control in environments that are not known a priori. Finally, conclusions end the paper.

II. MULTIPLE MODEL SWITCHED ADAPTIVE CONTROL STRUCTURE

The switched multiple model adaptive control (SMMAC) structure is shown in Fig. 1. Apart from the integrator (block Int) included to achieve zero tracking error for constant setpoints in the presence of constant disturbances, it is made of two main parts: A bank of candidate local controllers and a supervisor deciding which controller is to be active at each time. The bank of controllers $C_i$ for $i = 1, \ldots , n$
is designed such that each controller stabilizes and meets the desired performance specifications when applied to the corresponding nominal model $M_i$. The supervisor comprises a bank of nominal models $M_i$ for $i = 1, \ldots, n$, each one corresponding to a different set of plant models centered around an a priori known nominal plant model and a Performance Evaluator and Switching Logic block. The task of this block is to select the $M_i$ that has the dynamic behavior closest to the Plant and then apply the corresponding controller $C_i$. This is achieved by comparing a performance index $J_i(t)$, based on the output errors $(y(t) - \hat{y}_i(t))$ of each model $M_i$ for $i = 1, \ldots, n$, and choosing the controller which corresponds to the minimum at every instant. A discrete equivalent of the performance index used in [2] is given by,

$$J_i(t) = \alpha \varepsilon_i^2(t) + \beta \sum_{j=0}^{t-1} e^{-\lambda_P(t-j)} \varepsilon_i^2(t),$$

where $\varepsilon_i(t) = y(t) - \hat{y}_i(t)$ is the estimator error for the $i$ model and $\alpha$, $\beta$, are weighting factors of the terms that incorporate instantaneous and long-term measures of accuracy. The forgetting factor $\lambda_P$ determine the memory of the index. To avoid excessive and inappropriate switching due to outliers that may occur in the estimation error, a hysteresis dead-zone is defined so that switching takes place only if,

$$J_a(t) > (1 + h) \min_i J_i(t),$$

where $h > 0$, the hysteresis factor, is a design parameter and $J_a(t)$ is the actual controller performance index. Alternatively, a dwell time strategy may also be applied.

There are several combinations of fixed and adaptive models and the corresponding controllers to the structure discussed thus far. For a thoroughly discussion see [16].

III. Multiple Model Switched Adaptive Control with Model Learning

The main goal of the learning mechanism is, on the basis of on-line data, to add new nominal models and the corresponding controllers to the sets of nominal plant models and controllers, already in place, if they are considered good and useful according to criteria explained below.

Whenever the operating conditions are, in some way, different from those a priori known (represented by the set of nominal models), a new model is identified and a corresponding new controller is obtained, both being added to the nominal plant models and controllers sets. If at a later time the plant returns to that operating point, and the correct controller is chosen, an improvement in the transient response is expected.

As can be seen in Fig. 2, the learning mechanism introduces the following new functionalities,

- An on-line Identification algorithm;
- An on-line controller design algorithm;
- A decision that a new model must be added

If the SMMAC structure already includes at least one adaptive model/controller (as in the simulations below), the first two functionalities are no longer needed.

An identified model and the corresponding controller are added to the nominal models/controllers bank only when the following conditions are fulfilled:

1) The identified models are validated by a statistic test (to be explained below);

Fig. 1. Block diagram of multiple model switched adaptive control.
2) The identified models are different from the nominal ones;
3) The identified models perform better than the nominal models;

To validate the identified models, two types of statistical tests can be performed [17]. If identification methods based on the whitening of the prediction error (such as RLS) are used, then a whitening test on the residual prediction error sequence should be made. If identification methods, based on the decorrelation between the observations and the prediction errors (such as output error methods) are used, then an uncorrelation test between the residual prediction error and the delayed prediction model output sequences should be made. Additional details about these model validation tests can be found in [17].

Since, only behaviours corresponding to significantly different dynamics with respect to the already existing nominal models should give rise to new nominal models, a model is added only when the Vinnicombe distance [18] between the new model and the closest model, already in the model bank, exceeds a given threshold. This metric represents a measure of nearness between transfer functions and is defined, for single-input single-output systems as,

\[ \delta_v(P_1, P_2) = \sup_{\omega} \frac{|P_1(j\omega) - P_2(j\omega)|}{\sqrt{1 + P_1(j\omega)} \sqrt{1 + P_2(j\omega)}}, \]  

provided the winding number condition is satisfied (see [18] and [19] for further details). The \( P_i \) for \( i = 1, 2 \) in (3) represent transfer functions.

The last criterion that must be fulfilled before the acceptance of a new model/controller concerns the new model performance. It only makes sense to add up a new model when this one outperforms all the other existing nominal models. As a decision criteria for the new model, (2) is used. The performance index parameters values and the \( h \) value may be different from the ones already chosen to the nominal models.

The on-line design of the new local controller may be done by using a design rule. Alternatively, it is possible to explore the adaptive controller channel, if there is any (as in the example below), since it provides the gains for a controller that matches the newly identified model. In this latter case, an additional test for the convergence of the self-tuning controller gains has to be included.

IV. LOCAL CONTROLLER DESIGN

In this paper the MUSMAR adaptive control algorithm was chosen for the self-tuning channel and to design the SMMAC fixed controllers. In what concerns the known \textit{a priori} fixed models, the algorithm is applied off-line to each one of these models and run till convergence is obtained. The resulting controller gains are then recorded, defining the fixed controllers \( C_i \), that are thus good approximations to LQ controllers constrained to the selected controller structure.

This design aims at minimizing the quadratic cost functional defined over a \( T \) steps horizon:

\[ J_T(t) = \frac{1}{T} E \left[ \sum_{k=t}^{t+T-1} \hat{y}^2(k + 1) + \rho u^2(k) |I|^r \right], \]

where \( \rho \geq 0 \) is a penalty on the control effort, \( \hat{y} \) is the tracking error, \( \hat{y} = y(t) - y^*(t) \) and \( E[|I|^r] \) denotes the mean conditioned on the information available up to discrete time \( t \) (projection). Let the input to the integrator, \( u(t) \), be given by

\[ u(t) = F^t s(t) + \eta(t), \]

where \( F \) is the vector of controller gains minimizing the cost functional \( J_T(t) \) and \( \eta \) is a white dither noise of small amplitude. The vector \( s(t) \) is a sufficient statistic for
computing the control and is given by
\[ s(t) = [\tilde{y}(t) \ldots \tilde{y}(t-n+1) u(t-1) \ldots u(t-m)]' \quad (6) \]
The vector of gains is yielded by performing the following steps at each discrete time \( t \):

**MUSMAR Algorithm [9]**

1. For \( i = 1, \ldots, T \), compute the Recursive Least Squares (RLS) estimate of the parameters \( \theta_i, \psi_i, \mu_i \) and \( \phi_i \) in the following predictive regression models
   \[ \tilde{y}(t + i - T) \approx \theta_i u(t - T) + \psi_i s(t - T) \quad (7) \]
   \[ u(t + i - T - 1) \approx \mu_{i-1} u(t - T) + \phi_{i-1} s(t - T) \quad (8) \]
   where \( \approx \) denotes equality in least squares sense.

2. Update the controller gains according to
   \[ F = -\sum_{i=1}^{T} \theta_i \psi_i + \rho \sum_{i=1}^{T-1} \mu_i \phi_i \]
   \[ \sum_{i=1}^{T} \theta_i^2 + \rho(1 + \sum_{i=1}^{T-1} \mu_i^2) \]
   (9)

The parameters \( \theta_j, \psi_j, \mu_i \) and \( \phi_i \) in (7) and (8) can be computed from an underlying ARX model. In practice, it turns better to estimate them directly from plant data as a RLS projection.

A. Example

This example illustrates the tuning capabilities of MUSMAR in the presence of a plant with sparse abrupt changes. Consider the ARMAX model

\[ y_{t+2} + a_1 y_{t+1} + a_2 y_t = b_0 u_{t+1} + e_{t+2} + c_1 e_{t+1} \]

where \( u \) is the manipulated variable, \( y \) the measured output, \( e \) a sequence of independent, identically distributed random variables with zero mean and unit variance and \( t \) is the time. Three different systems \( \Sigma_1, \Sigma_2 \) and \( \Sigma_3 \) are defined as shown in Table I The system is coupled to the reduced

<table>
<thead>
<tr>
<th>System</th>
<th>( a_1 )</th>
<th>( a_2 )</th>
<th>( b_0 )</th>
<th>( c_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Sigma_1 )</td>
<td>0.9</td>
<td>0.95</td>
<td>1</td>
<td>0.715</td>
</tr>
<tr>
<td>( \Sigma_2 )</td>
<td>0.9</td>
<td>-0.6</td>
<td>1</td>
<td>0.715</td>
</tr>
<tr>
<td>( \Sigma_3 )</td>
<td>0.9</td>
<td>-1</td>
<td>0.5</td>
<td>0.715</td>
</tr>
</tbody>
</table>

complexity controller defined by

\[ u_t = f_1 y_t + f_2 u_{t-1} \]

Fig. 3 shows the level curves of the quadratic cost

\[ J_{LQ} = E[y_t^2 + \rho u_t^2] \]

with \( \rho = 1 \), for \( \Sigma_1, \Sigma_2 \) and \( \Sigma_3 \), as a function of the controllers gains \( f_1 \) and \( f_2 \), together with the respective stability boundaries for this controller.

A simulation is performed in which \( \Sigma_1, \Sigma_2 \) and \( \Sigma_3 \) are each one kept constant for 8000 samples and then switched to another \( \Sigma_i \), with \( f_1 \) and \( f_2 \) updated by MUSMAR. The dots represent the gains yielded by MUSMAR. With \( \Sigma_1 \), MUSMAR converged to the point indicated as \( E_1 \). When \( \Sigma_2 \) switches to \( \Sigma_2 \), the gains are initialized at \( S_2 \) (equal to \( E_1 \)) and converge to \( E_2 \), again close to the optimum LQ gains.

A similar situation happens when \( E_2 \) switches to \( \Sigma_3 \). It is remarked that, in all cases, MUSMAR is able to converge even when initialized with a non-stabilizing controller.

This example illustrates how MUSMAR may be used to yield controllers to \( \Sigma_1, \Sigma_2 \) and \( \Sigma_3 \) in succession.

V. DESCRIPTION OF THE FLEXIBLE TRANSMISSION SYSTEM

The control of a flexible transmission system was proposed in [13] as a benchmark problem. The system consists of three horizontal pulleys connected by two elastic belts, the objective being to position the angle of the third pulley (\( P_3 \)) by applying a torque to the first one (\( P_1 \)) (Fig. 4).

![Diagram of the Grenoble flexible transmission system.](image)

By varying the load on \( P_3 \), parameters changes are induced in the plant model. This motivated the use of several methods of robust control. In [20] the Horowitz design method was successfully used, although leading to high order controllers. Reference [21] follows an approach based on complex non-integer integration. Robust digital control using the combined pole placement /sensitivity function shaping method resulting in an RST structure is described in [22] in relation to this application. \( H_{\infty} \) methods are considered in [23] and [24] while [25] works with closed loop data from the plant and iteratively improves the performance. Constrained receding horizon predictive control, also resulting in an RST controller is described in [26]. While the above approaches aim at designing a controller that remains constant in time, the advantages of using an adaptive controller were recognized in [14], [15] where SMMAC was used and a detailed study on the influence of each design parameters was performed. Hereafter it is shown how to enlarge on-line the models/controllers bank using the procedure described in section III.

The system input \( u(t) \) is the angular position of the first pulley (\( P_1 \)), while the output \( y(t) \) is the angular position of the third pulley (\( P_3 \)). The control objective was to make
the angular position of the third pulley ($P_3$), which may be loaded with small disks, as close as possible, to a given reference signal $r(t)$ (tracking control problem). Three plant models corresponding to three different loads are available. They are described by the discrete transfer functions having the following structure:

$$H(q^{-1}) = \frac{q^{-d}B(q^{-1})}{A(q^{-1})} = \frac{q^{-d} \sum_{j=1}^{m} b_j q^{-j}}{1 + \sum_{i=1}^{n} a_i q^{-i}}, \quad (10)$$

with $m = 2$, $n = 4$ and $d = 2$. The parameters of the three identified models with a sampling frequency of $20\,Hz$ are given in Table II.

The main characteristics of this system are the low damped vibration modes (damping factors less than 0.05) and significant change of resonance frequencies with load. A variation of about 100% of the frequency of the first vibration mode occurs when passing from the full loaded case to the unloaded case.

In the simulations that follow the dynamics of the transmission depend on the load and available were discrete-time ARX models for the three different load cases,

$$A_k(q^{-1})y(t) = B_k(q^{-1})u(t) + e(t), \quad k = NL, HL, FL, \quad (11)$$

with $e(t)$, the disturbances acting on the plant, a zero mean white sequence of identically distributed gaussian random variables with variance $\sigma^2 = (0.01)^2$.

VI. Simulation Results

In this section, resorting to simulations, the proposed Switched Multiple Model Adaptive Controllers (SMMAC), with and without learning, are analyzed and compared to each other for the flexible transmission system.

The objective of the control system is to track the output of a filter $F(s) = 5/(s+5)$ to a square wave reference input, with unit amplitude and period $T_s = 100\,s$, for a system whose dynamic behavior changes abruptly in time. Thus, in order to compare different control structures a performance index is defined as follows:

$$J_C(t) = \frac{1}{T} \sum_{k=0}^{T} (\tilde{y}^2(k) + u^2(k)), \quad (12)$$

This index measures the discrepancies between the real output of the plant $y(t)$ and the desired one $y^*(t)$ as well as the control signal.

The SMMAC structure has one fixed and one adaptive model/controller with reinitialization. It is assumed that only the No Load (NL) model is initially known. The adaptive MUSMAR algorithm used in the self-tuning channel and

<table>
<thead>
<tr>
<th>TABLE II</th>
<th>THE THREE MODELS PARAMETERS</th>
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<tbody>
<tr>
<td></td>
<td>No Load (NL)</td>
</tr>
<tr>
<td>$a_1$</td>
<td>-1.35277</td>
</tr>
<tr>
<td>$a_2$</td>
<td>1.55021</td>
</tr>
<tr>
<td>$a_3$</td>
<td>-1.27978</td>
</tr>
<tr>
<td>$a_4$</td>
<td>0.91147</td>
</tr>
<tr>
<td>$b_1$</td>
<td>0.41156</td>
</tr>
<tr>
<td>$b_2$</td>
<td>0.52397</td>
</tr>
</tbody>
</table>
to generate the fixed controller, that corresponds to the NL model (the active controller 1 of Fig. 5 and 8), has the following parameters $T = 30$, $n = 3$, $m = 2$, $\lambda = 0.998$, $p = 1$, $P_0 = 1000$ and $\sigma^2_\eta = 0.00001$.

The design parameters for the supervisor performance index are $\lambda_P = 0.1$, $h = 1.6$, $\alpha = \beta = 1$.

In both simulations that are described hereafter the plant experiences the same time variations as depicted in the top of Fig. 5 or Fig. 8.

A. Simulation 1

In this first simulation, whose results can be seen in Figs. 5 up to 7, the SMMAC structure was tested without learning. Fig. 5 shows two graphics. The one above shows the actual model of the plant as a function of time. Below, it shows the local controller in use (i.e. applied to the plant). This may be either controller 1, that matches the situation of No Load (NL) or the self-tuner (MUSMAR) when the supervisor decides that model 1 does not match the plant dynamics. Actually, in situations of Full Load (FL) or Half Load (HL) the self-tuner is (correctly) selected. Despite the supervisor good behavior (only two false detections; The worse correct detection happened 72 samples after the plant dynamics change), there are very strong transients at the instants in which the plant changes from NL or HL to FL. These results were expected since there is a notorious lack of fixed models, due to a former plant insufficient knowledge, in the SMMAC structure. Although, the adaptive controller shows a very good steady state reference following, whenever a large dynamics change takes place the controller reaction is slow and a poor transient response occurs.

B. Simulation 2

To overcome the insufficient plant knowledge, a learning mechanism was embedded in the SMMAC structure. For identical simulation conditions this section reports the results obtained with this new structure (Figs. 8 - 10). For identifying new local models, RLS with exponential forgetting [9], [17] was used. This choice was based on an a priori knowledge of the plant, whose dynamics changes have an ARX structure. The RLS with exponential forgetting is initialized with its parameters equal to zero $\hat{\theta}(0) = [0 \cdots 0]$, the covariance matrix $P(0) = 1000I$ and a forgetting factor of 0.997. The Whitening test was chosen, as the model validation test (section III). In what concerns the similarity between plants, it is assumed that two systems are different whenever the $\nu$ gap distance between them is greater than 0.3.

The simulation is started with the same adaptive and fixed models as in the previous one, a fact that justifies the output signal and control signal behavior in the first 300 seconds of both simulations. As shown in Fig. 8, during this period two new models/controllers were learned (corresponding to the situations of HL and FL) and added to the bank. As it happens, when the load returns to any of these values in future situations, these new controllers will be selected.
instead of the self-tuner. The last 600s of simulation (time interval where all the dynamics behavior is repeated), show an improvement in the performance of this new structure, particularly in regions where the system become full Loaded, i.e., at 400s and 700s (compare Fig. 7 and 10). The better transients are followed closely by less control effort (compare Fig. 7 and 10).

The active controller of Fig. 8 shows the best model chosen by the supervisor at each instant. In this diagram Adapt corresponds to the adaptive controller and 1, 2 and 3 are fixed controllers that correspond, respectively, to NL, FL, and HL. The model NL is already known before the simulation and the controllers 2 and 3, correspond to additional fixed models added during the simulation at the instants marked by the arrows in the figure.

The new switching scheme shows better performance in the input and output response compared to the conventional adaptive control with multiple models. Compare Figs. 7 and 10 for the tracking behaviour and Figs. 6 and 9 for the manipulated variable. The superior behavior is, of course, due to the enlargement of the model bank. After including in it the models/controllers corresponding to FL and HL, the SMMAC no longer needs to rely on the self-tuner when these situations are found, and this results in the suppression of the corresponding adaptation transients.

VII. CONCLUSIONS AND FUTURE WORK

A. Conclusions

A new Multiple Model Switched Adaptive Control structure with learning for improving the transient response of the adaptive control using multiple models is proposed and illustrated through the application to a flexible transmission system. The main idea is to add a new fixed model/controller whenever it is considered appropriate according to some criteria. This approach is especially useful whenever the previous plant knowledge is insufficient and the plant dynamics exhibits abrupt changes.

Based on the simulation results of the flexible transmission position control the SMMAC with and without learning, were compared to each other. From the use of learning resulted a clear advantage to the system’s closed-loop dynamical behaviour, specifically, when repeatedly returning to the same operating point that was not envisaged in the initial bank of local controllers.

The issue of deleting models/controllers from the bank may also be considered. Having too many local controllers may result in performance degradation. According to the approach followed, the local controllers that are added correspond to actually observed plant dynamic modes. This, and the condition based on the Vinnicombe metric prevents unnecessary controllers to be added, at least from the point of view of stability. However, this is a matter to further elaborate.

REFERENCES


