AN AUTOPILOT FOR A SMALL AIRCRAFT: EXPLORING
ADAPTIVE CONTROL

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Abstract: This paper is concerned with the project of an automatic flight control system for an uninhabited aircraft of small dimensions. The work starts by designing the structure of the control system including the definition of the different control loops and their interconnections. Within this structure, local linear controllers are designed for the different possible operating points and then integrated in a gain scheduling architecture. This approach is confronted with the use of the predictive adaptive MUSMAR control algorithm.

Keywords: Aircraft, Local Linear Controllers, Adaptive Control, MUSMAR, Gain Scheduling.

1. INTRODUCTION.

With the constant development of the aviation industry, there are many, commercial and military, aircraft being developed and built for several applications.

One of the most active areas in the development of airplanes is of UAVs - Uninhabited Aerial Vehicles, where currently 32 nations are developing and constructing over 250 models. Its utility is sufficiently well-known in diverse applications such as military (e.g. recognition, control of borders, etc.) and civil (e.g. rescue search, forest and traffic control, etc.). Even NASA (National Aeronautics and Space Administration) is developing UAVs to allow atmospheric studies.

As reported in the recent reviews (Balas et al., 2003), several different approaches for designing flight control laws may be considered. This paper presents one possible approach, relying on local controllers suitably patched, to the control of an UAV.

1.1 Problem formulation.

The model of the considered aircraft is obtained from a 1/4 scale Piper PA 18 Super Cub equipped with an engine of 50 cc (Rato et al., 1993). This model is implemented in Simulink being validated through the study of the aircraft own oscillatory modes.

The dynamic of an aircraft is dependent on the airspeed, localization of the centre of mass and altitude. It is assumed that the localization of the centre of mass is constant and that the altitude variations are very small so that the dynamics are only dependent on the airspeed of the aircraft.

In order to solve the dynamics variations with the airspeed of the aircraft (non-linearities), which affects the control of the system, two alternative strategies are adopted - integration of linear local controllers in a gain scheduling architecture and adaptive techniques (MUSMAR).

1.2 Paper contributions and organization.

The contributions of this paper consists in a case study on the design of an automatic flight control system, including the design of local linear controllers using polynomial techniques, the integration of the controllers in a gain scheduling architecture and the development of adaptive controllers.

The paper has the following structure: After introducing the problem and having focused on the methods used for its solution (present section), the structure
for the control system is described on section 2. The
design of the local linear controllers are presented on
section 3, where the identification of the necessary
chains for control is also included. The integration of
these controllers in a gain scheduling architecture is
presented in section 4. Section 5 presents the study of
adaptive techniques, MUSMAR algorithm, while on
section 6 the main conclusions are drawn.

2. STRUCTURE OF THE CONTROL SYSTEM.

For the control loops, required by the most usual
maneuvers of aircraft, the variables chosen are:
- Pitch, \( \theta \).
- Roll, \( \phi \).
- Yaw, \( \psi \).
- Longitudinal Velocity (Airspeed), \( U \).

With these variables a pilot or a guiding system can control
the trajectory of the aircraft.

The aircraft has 5 actuators: ailerons \( (\eta_{a}) \), flaps \( (\eta_{f}) \),
rudder \( (\eta_{r}) \), elevators \( (\eta_{e}) \) and throttle \( (T_{H}) \).

It is remarked that different control structures can perform
the desired tasks.

Since the flaps are mainly used to increase the maximum
sustentation during landing and taking off, they were not considered for the control system, remaining
in the rest position. In what follows the above manoeuvres are not considered.

2.1 Longitudinal control.

The engine is used for airspeed control, while the
elevators are used for pitch control. In both cases the
variables are regulated by feedback loops designed
individually. These structures are presented on figures
1 and 2.

Fig. 1. Structure of pitch control.

Fig. 2. Structure of airspeed control.

Another form for the structure of longitudinal control
would be to control the airspeed with the elevators and
the pitch with the engine. However this choice shows
more limitations than the structure adopted.

2.2 Lateral control.

Lateral control is important so that the aircraft is able
to make turns. The ailerons, when compared with the
rudder, allow more efficient turns (smaller turn radius)
because, in this case, the aircraft leans laterally (roll angle)
in the direction of the rotation which has as consequence a variation in the angle \( \psi \).

The use of the rudder is also necessary so that the
craft can perform coordinate turns, or in order to
prevent horizontal slide, so the angle of sideslip \( (\beta) \)
remains null.

The structures for the lateral control can be seen in
figures 3 and 4.

Fig. 3. Structure of turns control.

Fig. 4. Structure of sideslip control.

3. LOCAL LINEAR CONTROLLERS.

Having defined the control system structure, the project of local linear controllers is now performed.
For that sake it is necessary to identify the transfer
functions of the chains referred previously.

The controllers will be designed in the discrete time
domain.

3.1 Sampling Period and Identification.

In the choice of the sampling period factors have to be
taken in to account (Åström et al., 1989). In this case
the sampling period was defined as 0,1s.

Since the aircraft is a MIMO (Multiple Input Multiple
Output) system, and the outputs are not affected only
by one input, the identification of each chain can not
be done neglecting the other chains. So the identification
process assumes a MISO (Multiple Input Single
Output) structure. Due to the interference between
loops it is advisable to assume the presence of colored
noise, so the structure considered for the models is
ARMAX and the identification method chosen was
the Maximum Likelihood. The signal chosen for the
identification was a PRBS. In order to obtain the best
possible models some cares were taken, such as: re-
moving the linear trends and use of prediction error
for validation.
According to the aircraft airspeed range, four regions were defined: low velocities (18 m/s), medium-low velocities (22 m/s), medium-high velocities (26 m/s) and high velocities (30 m/s). The models for the various working points were identified.

3.2 Polynomial control.

The project for the local linear controllers was based on polynomial controllers with two degrees of freedom as presented in figure 5. (Åström et al., 1990)

\[ T \rightarrow R \oplus u \rightarrow B \rightarrow A \rightarrow d \]

Fig. 5. Polynomial controllers structure.

The process is modelled by the transfer function, \( H(z) = B(z)/A(z) \), and we want to find a controller (polynomials \( R, S \in T \)) such that the closed loop controller behaves as \( H(z) = B_m(z)/A_m(z) \) (desired transfer function).

The problem consists on finding \( R, S \in T \) such that, in close loop,

\[ \frac{BT}{AR + BS} = \frac{B_m A_o}{A_m A_o} \quad (1) \]

where \( A_o \) represents the polynomial for the observer.

The desired transfer functions were defined as responses to second order systems. In the case of the system having more than two poles, the dominant response would be of the second order system and extra poles were chosen close to the center of the unit circle in order to maintain the global response. The specifications for the controllers were defined by the establishing time and overshoot of the desired transfer function.

Controllers for all the loops and working points were developed and tested with success. In the figure 6 the result for the airspeed may be seen.

All of the specifications are obeyed and the response of both the linear (identified) and non-linear system are very similar, which implies that the identified models have retained the main characteristics of the non-linear system. The response of the controllers in the presence of turbulence was also studied using the Dryden model to simulate turbulence (McLean, 1990), figure 7.

As can be seen, the oscillations caused by the turbulence are rejected and the controller allows the aircraft to keep stable and follow the desired references.

![Fig. 6.airspeed control](image)

**Fig. 6. Airspeed control.**

![Fig. 7.airspeed control in the presence of turbulence](image)

**Fig. 7. Airspeed control in the presence of turbulence.**

4. GAIN SCHEDULING.

In order to make the system’s dynamic response independent of the working point, gain scheduling techniques are commonly used. These consist of making the parameters of the controller vary with the working point in a pre-programmed way depending on a "selection variable". In this paper the goal is to make the system independent of its airspeed.

There are a certain number of conditions that, in closed loop, guarantee global stabilization (Johansen et al., 1997).

The gain scheduling architecture was tested on the airspeed, pitch and roll chains individually in order to avoid interferences between them.

Figure 8 shows the result for the airspeed. The first graphic shows the response with the gain scheduling architecture. All the required specifications are met. The second graphic shows the result when a single controller is used. As it can be seen, the system remains stable and references are followed, which implies that a single controller would be enough for this chain. On the third graphic is seen the influence of the pitch on the airspeed, where in this loop a single controller is used. As can be seen, for higher velocities some oscillations can be observed.
of the required specifications being met and in all cases it shows superior performances to when a single controller is used.

5. ADAPTIVE CONTROL - MUSMAR.

The gain scheduling technique (section 4), handles the problems related with slow variations of airspeed and/or altitude. However unpredictable variations of dynamic pressure and centre of mass on the aircraft cannot be solved by gain scheduling. So an adaptive algorithm is studied - MUSMAR.

5.1 MUSMAR principles.

In MUSMAR the manipulated variable is selected such as to minimize the multi step quadratic cost functional:

$$ J \triangleq \frac{1}{T} E \left[ \sum_{i=1}^{T} [\dot{y}^2(t+i) + \rho u^2(t + i - 1)]|I(t)| \right] $$

where:

- $\dot{y}(t+i) \triangleq y(t + i) - r(t + i)$.
- $y(t)$, $r(t)$ are respectively the output, the input and the reference for the process.
- $T$ is the horizon of control.
- $\rho$ its a penalization factor of the control action.
- $E(|I|)$ represents the operator mean value conditioned to the information $I(t)$ obtained from the observations of $y$ and $u$ until the instant $t$.

The predictive models used by MUSMAR are defined as follows (Zobrist et al., 1994):

$$ \dot{y}(t + i) = \psi_i u(t) + \psi'_i s(t) + \nu'_i(t) $$

$$ u(t + i - 1) = \mu_i - 1 u(t) + \phi'_i s(t) + \nu''_i(t) $$

for $i = 1, ..., T$

Here $s(t)$ is the pseudostate vector - vector with the necessary past information required for the computation of the manipulated variable.

The minimization of 2, in respect to $u(t)$, using 3 and 4, yields:

$$ u(t) = \frac{\sum_{i=1}^{T} \theta_i \psi_i + \rho \sum_{i=1}^{T-1} \mu_i \phi'_i}{\sum_{i=1}^{T} \theta_i^2 + \rho (1 + \sum_{i=1}^{T} \mu_i^2)} s(t) $$

To the control law in (5) a disturbance signal is added (dither), $\eta(t)$, to guarantee a persistent excitation condition.

The parameters $\theta_i$, $\psi'_i$, $\mu_i$, $\phi'_i$, of the models (3) and (4) which are necessary for (5), are estimated at every sampling instance using an identification algorithm.

Fig. 8. Gain scheduling - airspeed.
Similar studies were then made for the pitch and roll, figures 9 and 10.

The advantage of using gain scheduling (GS) is readily seen by comparing the second and third graphics of figure 9. Without gain scheduling (i.e., with a single controller - third graphic of figure 9) the pitch enters into oscillation when the airspeed of the aircraft is high.

Fig. 9. Gain scheduling - pitch.

From the results presented it is concluded that the system with gain scheduling remains stable with all
namely the Directional Forgetting Recursive Least Squares algorithm (DFRLS) (Zobrist, 1994).

The use of the MUSMAR algorithm on the pitch chain, considering in the remaining chains the local controllers developed earlier (section 3) is considered hereafter.

5.2 Pitch control.

In order to have a null position static error, an integrator was included in the loop. Thus the block diagram is as shown in figure 11:

![Fig. 11. MUSMAR with integrator structure.](image)

An equivalent form to the control law, is:

\[ R(q)u(t) = T(q)ref(t) + S(q)\hat{y}(t) \]  \hspace{1cm} (6)

In stationary regimen \( u_i \) (input process signal) is constant, so at the integrators input one has \( u = 0 \). This means that, in stationary regimen:

\[ \hat{y} = \frac{T(q)}{S(q)} ref \]  \hspace{1cm} (7)

For a null tracking error it is necessary that \( T(q) = 0 \) or \( ref = 0 \) (possibility that restricts the references which allow a static error 0).

In order to prove this necessary condition, \( T(q) = 0 \), two simulations were carried: one with a reference in the pseudostate and another one without. (figures 12 and 13)

![Fig. 12. MUSMAR with integrator applied on the chain of action and with a reference in the pseudostate. Gains, tracking and elevator.](image)

![Fig. 13. MUSMAR without a reference in the pseudostate. Gains, tracking and elevator.](image)

It can be concluded that the anticipation effect of the reference and the integral effect may compete, leading to an error which is not null, that is not desired.

The results, presented on figure 13, are the expected ones concerning the static error (null). However the overshoot is excessive.

A possibility to improve these responses is to add extra-information to the pseudostate vector. The most logic is to include information about the variables derivative, which for the pitch loop is the rotational airspeed \( Q = \dot{\theta} \). (figure 14)

![Fig. 14. MUSMAR with rotational airspeed in the pseudostate. Gains, tracking and elevator.](image)

As it can be seen in figure 14 the results have improved considerably, since the overshoot is now about 2.2% with an establishing time of 1.5s.

So far, the airspeed has been considered constant. However the interest on adaptive controllers appears when the dynamics varies. Therefore the result of the MUSMAR control when the airspeed varies is presented (figure 15).

As can be seen the quality of the response is very dependent on the penalty for the control action. In this case, \( \rho \) was tuned for lower velocities, which prove-
Fig. 15. MUSMAR with variable airspeed. Airspeed
and tracking.

To solve this problem the penalty action was made
variable with the airspeed, by means of a first order
interpolation, considering the cited extremities. On
figure 16 the qualitative improvement of the results is
visible when \( \rho \) depends on \( U \).

Fig. 16. MUSMAR with variable airspeed and variable
\( \rho \). Airspeed and tracking.

6. CONCLUSIONS.

The problem of an automatic flight control system
has been considered and solved. In order to control
the aircraft a flight control system was developed
considering the control variables: roll, yaw, pitch and
airspeed.

The project of the local controllers was based on
polynomial control, this local linear controllers being
designed for each flight condition. For the design of
the local controller network one needs to know the
transfer functions for which an identification process
is necessary. The identification of each loop could
not be done neglecting the others since there is an
interference between them. The controllers developed
showed good behavior on the nonlinear system and
also when they were facing disturbances.

With the intuition of making the system performance
independent of airspeed, gain scheduling techniques
were used. In all the chains tested, gain scheduling
revealed improved performances with respect to a
single controller.

In order to solve unpredictable variations of dynamic
pressure and centre of mass or malfunction on the
aircraft an adaptive algorithm was studied - MUSMAR.
When applied to the pitch chain, it was possible to
conclude that the competition between anticipation
effect of the reference and the integral effect can lead
to a non null static error position. Finally to solve
the quality tracking dependence with \( \rho \) when airspeed
varies, a first order interpolation was made making \( \rho \)
dependent with airspeed.

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